# Kähler identites for almost complex manifolds

Luis Fernandez
Bronx CC and Graduate Center, City University of New York, USA
lfernandez1@gc.cuny.edu

On a Kähler manifold, the Kähler identities (also called Hodge identites) give a commutator relationship between some naturally defined operators, namely $d \circ \Lambda - \Lambda \circ d = d^{c*}$, where $d$ is the exterior derivative, $\Lambda$ is the adjoint of the Lefschetz operator $L$ (defined by wedging with the fundamental form $\omega$), and $d^c$ is defined by conjugating $d$ with the complex structure $J$. They are a fundamental tool to prove, for example, the equivalence of different Laplacians or the Lefschetz decomposition, which leads to essential topological restrictions for Kähler manifolds.

There are generalizations of these identities for complex manifolds by Demailly, for almost Kähler manifolds by Bartolomeis and Tomassini, and for nearly Kähler manifolds by Verbitsky.

In this talk we will give a complete generalization, for almost complex manifolds, of the Kähler identities, and will find several other insteresting commutator relations. This generalization was also found, independently and using different methods, in an unpublished work by de la Ossa, Karigiannis and Svanes.

The main technique is to prove a version of these identities in the Clifford bundle, using the Dirac operator instead of $d$, and then translate it to the exterior bundle. The method of using the Clifford bundle to study these operator, which to my knowledge appeared originally in [2], has an interest on its own, and most probably has applications to other problems.

## Acknowledgements

Joint work with Samuel Hosmer.

## References

[1] L. Fernandez and S. Hosmer, *Kähler identities for almost complex manifolds*, https://arxiv.org/abs/2208.07276.

[2] M. L. Michelsohn, *Clifford and spinor cohomology of Kähler manifolds, Amer. J. Math.* ***102*** *(1980), no. 6, 1083–1146.*