On isoperimetric local-Bollobás-Thomason type inequalities

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Bollobás and Thomason proved a type of isoperimetric problem relating the volume of the projections of a convex body (a convex and compact set with non-empty interior in $\mathbb{R}^n$) onto coordinate subspaces and the volume of the projections of an appropriate coordinate box, which depends on K, onto the same subspaces.

In this talk we show the next isoperimetric type result which extends the one proved by Bollobás and Thomason. Given $K \subseteq \mathbb{R}^n$ a convex body and $\sigma \subset [n] := \{1, \dots, n\}$, there exists a suitable Hanner polytope B_K, which depends on K, for which

1. $\mathrm{vol}_n(K) = \mathrm{vol}_n(B_K)$,
2. $\mathrm{vol}_{n-|\sigma|+|\tau|}(P_{H_\sigma^\perp \oplus H_\tau} K) \geq \mathrm{vol}_{n-|\sigma|+|\tau|}(P_{H_\sigma^\perp \oplus H_\tau} B_K)$ for every $\tau \subseteq \sigma$, and
3. $\mathrm{vol}_{n-|\sigma|}(P_{H_\sigma^\perp} K) = \mathrm{vol}_{n-|\sigma|}(P_{H_\sigma^\perp} B_K)$.

Above $P_{H_\tau} K$ means the orthogonal projection of K onto the subspace $H_\tau := \langle e_i : i \in \tau \rangle$ for any $\tau \subseteq [n]$, with $\{e_i\}_{i=1}^n$ being the canonical basis of $\mathbb{R}^n$, and $\mathrm{vol}_n(\cdot)$ means the n-dimensional volume or the Lebesgue measure. Finally, we will see some functional results that will recover the previous one.

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References

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