

# A geometry of cubic discriminants in 8 dimensions

Ivan Minchev

Faculty of mathematics and informatics,  
Sofia University “St. Kliment Ohridski”, Bulgaria  
`minchev@fmi.uni-sofia.bg`

We consider the geometry of Riemannian 8-manifolds that admit a reduction of the structure group from  $SO(8)$  to the irreducible  $SO(4)$ . We show that such a reduction is determined by the existence of a special rank-4 tensor field on the manifold—called a cubic discriminant—that possesses the algebraic properties of a hyper-Kähler curvature and that is locally given by the formula for the discriminant of a cubic polynomial. Our approach resembles, in a way, the constructions related to the irreducible  $SO(3)$  geometry in dimension five, introduced by Bobieński and Nurowski [2].

Examples of cubic discriminants are provided by the quaternion-Kähler symmetric spaces  $M_+ = G_2/SO(4)$  and  $M_- = G_{2(2)}/SO(4)$ .  $M_+$  is one of the classical Wolf spaces with positive Ricci curvature, whereas  $M_-$  is a space with negative Ricci curvature that appears in the classification [1]. We show that  $M_+$  and  $M_-$  are locally the only integrable non-flat cubic discriminants. A new curvature characterization is given for the Riemannian metrics on these two exceptional symmetric spaces.

## Acknowledgements

Joint work with Elitza Hristova, Institute of Mathematics and Informatics, Bulgarian Academy of Sciences.

## References

- [1] D. V. Alekseevskii, *Classification of quaternionic spaces with a transitive solvable group of motions*, Math. USSR Izvestija, 9 (1975) 297–339.
- [2] M. Bobieński, P. Nurowski, *Irreducible  $SO(3)$  geometry in dimension five*, J. reine angew. Math., 605 (2007) 51–93.