

Null hypersurfaces and null cones in Lorentzian manifolds

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Hypersurfaces in a Lorentzian manifold are traditionally classified as timelike, spacelike or null. The latter case means that the induced metric tensor is degenerate everywhere on the hypersurface. This is why the study of null hypersurfaces is more difficult than the nondegenerate ones. Nevertheless, it is worth developing specific techniques to study them because null hypersurfaces are important from both a mathematical and a physical point of view.

The main technique to handle a null hypersurface involves the choice of some arbitrary geometric objects, as for example a null section, i.e. a vector field tangent and orthogonal to the null hypersurface. Once a null section have been fixed, a function on the null hypersurface called *surface gravity* can be defined. This function has a physical meaning and it changes in a nice way if we change the null section. This naturally raises the following question: Can we choose a null section such that its associated surface gravity is constant? In general, the answer is negative, but we can show that under some geometrical mild conditions we can ensure it. Moreover, we can also show that the existence of a null section with zero surface gravity imposes severe restriction on the null hypersurface. In fact, in this case, under some additional conditions, the null hypersurface is contained in a null cone.

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