Geometric and topological rigidity of pinched submanifolds

Theodoros Vlachos
Department of Mathematics, University of Ioannina, Greece
tvlachos@uoi.gr

We investigate the geometry and topology of compact submanifolds of arbitrary codimension in space forms, focusing on a specific pinching condition that relates the length of the second fundamental form to the mean curvature. Our findings demonstrate that this pinching condition either leads to the vanishing of homology in a range of intermediate dimensions or uniquely determines the submanifold up to congruence. These results are both sharp and generalize previous work by several authors, achieved without imposing additional assumptions on the mean curvature.

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