Curvature conditions underlying second-order superintegrable systems

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In this talk, I am going to discuss (irreducible) second-order (maximally) superintegrable Hamiltonian systems on Riemannian manifolds. Two prominent examples are the (non-degenerate) harmonic oscillator system and extensions of the Kepler-Coulomb system. Conformal geometry (or, more precisely speaking, Weylian geometry) naturally underpins such systems. I am going to present curvature conditions that apply to second-order superintegrable systems under various 'non-degeneration' conditions. For a large class of systems, the underlying manifold carries a Hessian structure, i.e. the metric is given by the Hessian of a function with respect to a certain natural flat torsion-free connection.

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