Volatility Modeling and Spillover: The Turkish and Russian Stock Markets

Ahmet Galip Gençyürek

Abstract
This study investigates the internal and external (spillover) characteristics of the volatility of the Turkish and Russian stock market indices. To this end, generalized autoregressive conditional heteroskedasticity models that are classified as short memory (GARCH, EGARCH, GJR-GARCH, APARCH) and long memory (FIGARCH, FIEGARCH, FIAPARCH, HYGARCH) considering adaptive structure (Fourier series), and the rolling Hong causality methods are used. The analysis spans the years 2003–2020, revealing that the asymmetric power autoregressive conditional heteroskedasticity model is the most appropriate method in terms of both stock indices and leverage and long memory effects are evident in the volatility series. Bidirectional volatility spillovers between Turkish and Russian stock market indices are also evident in all time horizons. Investors can use volatility results for stock valuation, risk management, portfolio diversification, and hedging, and policymakers can consider the volatility results to evaluate the fragility of financial markets.

Keywords
Stock Markets, Volatility, GARCH Models, Spillover, Time-Varying

Introduction
Volatility modeling and forecasting are crucial because volatility is considered a risk measure (Yu, 2002; Terasvirta, 2009). Higher volatility indicates riskier assets and reveals instability (Bhowmik, 2013). In addition, volatility reflects the fundamentals of the market as well as critical information (Ross, 1989) and market expectations (Kalotychou & Staikouras, 2009). Therefore, volatility modeling is essential for asset allocation, risk management, stock valuation, derivative valuation, and hedging (Pindyck, 2004; Ewing & Malik, 2017; Wu & Wang, 2020; Zhang et al., 2020; Lyocsa et al., 2021).

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The most frequently used models to measure volatility consider the conditional first moment. Returns on assets (first moment) have been assumed to be independent identically distributed (i.i.d.) random processes with zero mean and constant variance (Bollerslev et al., 1994; Xekalaki & Degiannakis, 2010); however, in the literature, variance has been reported as constant in the long-term, but changing during fluctuation periods (Kutlar & Torun, 2013). Studies by Engle (autoregressive conditional heteroskedasticity, ARCH) in 1982 and Bollerslev (generalized autoregressive conditional heteroskedasticity; GARCH) in 1986 considered these properties of time series and modeled the second moment (Chong et al., 1999). Since then, improvements have also been made to capture complex volatility dynamics (Brooks, 2007). Non-normal distribution, volatility clustering, the leverage effect, and long memory are the identified characteristics of the volatility of the financial time series (Carroll & Kearney, 2009). Volatility clustering indicates that “large changes tend to be followed by large changes —of either sign—and small changes tend to be followed by small changes” (Mandelbrot, 1963: 418), and indicates that volatility tends toward mean reversion (Bose, 2007). Volatility clustering and time series’ long memory properties are related. Long memory is a phenomenon observed in volatility clustering (Liu, 2000), which denotes a hyperbolic reduction in autocorrelations (Pong et al., 2008) and demonstrates that the market responds to the arrival of news slowly over time (Bentes, 2014). Therefore, researchers and investors can use past market movements to forecast future movement. Cognizance of the long memory property of time series helps to investigate the weak efficient market hypothesis, rejecting technical analysis. Heterogeneous information arrival and structural breaks may cause persistent behavior in volatility clustering (Andersen & Bollerslev, 1997). Liesenfeld (2001:173) noted that “short-run movements of volatility are driven by [the] news arrival process and the long-run movement of volatility by the sensitivity to news.” Empirical studies regarding volatility modeling show the volatility of asset prices to be considerably in persistent behavior. (e.g., Berger et al., 2009; Bentes et al., 2008; Baillie & Morana, 2009; Christensen et al., 2010; Chikli et al., 2012; Bentes, 2014; Nasr et al., 2016; Kuttu, 2018; Lahmiri & Bekiros, 2021). The leverage effect shows that “an unexpected drop in price (bad news) increases predictable volatility more than an unexpected increase in price (good news) of similar magnitude” (Engle & Ng, 1993: 1752). Christie (1982) indicated that financial leverage causes a leverage effect. A decrease in stock price elevates a firm’s leverage, and this circumstance enhances the risk borne by shareholders, and the expected stock return rises, along with the variance of stock return (French et al., 1987). A small number of researchers have explained the leverage phenomenon using volatility feedback. In cases of abundant positive news regarding the dividend policy, future volatility and expected return rises, and stock prices subsequently fall. In cases of abundant negative news regarding future dividend policy, stock price falls, as with the previous example and volatility and expected return increases, but this volatility deepens the negative effect of dividend policy and large negative stock returns are more prevalent, which can produce excess kurtosis (Campbell & Hentschel, 1992).
The features expressed (volatility clustering, leverage effect, and long memory) are obtained from analyses of the internal dynamics of volatility. This circumstance is referred to as a “heat wave” in the literature. According to this hypothesis, a shock in a market only affects the conditional volatility in that market (Engle et al., 1990). In addition, analyses of the external structure of volatility can also be conducted because volatility is affected by its own past fluctuations as well as fluctuations in other markets (Hong, 2001; Liu et al., 2017), which is referred to as a “meteor shower” (Engle et al., 1990). Two circumstances can cause meteor showers: trade and investment relationships and market psychology (Lin et al., 1994). The free flow of goods and capital and technological progress expedite the occurrence of meteor showers. Meteor shower (spillover) movement is consistent with the efficient market hypothesis (Koutmos & Booth, 1995). The research methods used to examine spillover include multivariate ARCH models (i.e., BEKK, VECH, CCC, DCC, cDCC, etc.), variance causality models (Cheung–Ng, Hong, Hafner–Herwartz), and spillover index models (Diebold Yilmaz, Barunik–Krehlik).

This study analyzes the properties of the univariate financial time series structure in Turkish and Russian stock market indices because both countries are considered emerging countries. From this perspective, the first aim is to determine the most relevant volatility structure of both countries’ leading stock market indices (BIST 100 and RTSI). Özden (2008), Kutlar & Torun (2012), Karabacak et al. (2014), Altuntas Taspunar & Colak (2015), Yıldız (2016), and Ay & Gün (2020) demonstrated that TGARCH is the most suitable ARCH model for stock market indices (e.g., BIST 100 - financial and industrial index-banking) in Türkiye; however, these studies did not consider long memory models in their studies. Koy & Ekim (2016) and Topaloğlu (2020) argued that GARCH or EGARCH are the most appropriate models for stock market indices according to the indices (e.g., industry, trade, services, banking, and financial) analyzed in Türkiye. Cevik (2012), Cevik & Topaloglu (2014), Günay (2014), Gaye Gencer & Demiralay (2015), Büberkökü & Kizildere (2017), and Celik & Kaya (2018) revealed a long memory in conditional variance, asserting that and fractional integrated ARCH models are suitable for modeling volatility in stock indices. In addition, Kula & Baykut (2017) demonstrated that the GARCH model is the most appropriate model for BIST and RTSI among short memory models. Reviewing the results obtained in previous literature, it is apparent that the most suitable ARCH model depends on the methods used, the sample, and the time examined; however, volatility generally demonstrates long memory properties, which appears to reject the weak efficient market hypothesis.

The pairwise relationship between stock market index volatility in both countries has been also investigated because Türkiye and Russia are neighboring countries on the shores of the Black Sea and are highly interconnected, both politically and economically. In 2020, Türkiye’s exports to Russia amounted to 4.4 billion dollars, while Russia’s exports to Türkiye amounted to 17.8 billion dollars (Foreign Economic Relations Board, 2021). Yarovaya et al.
(2016), Dedi & Yavas (2016), Gökbülut (2017), Bayramoğlu & Abasız (2017), Kocaarslan et al. (2017), Berberoglu (2020), and Kutlu & Karakaya (2020) analyzed volatility spillover between BIST 100 and RTSI with differing results. Yarovaya et al. (2016) and Gökbülut (2017) found a bidirectional interaction in volatility; however, Yarovaya et al. (2016) demonstrated that the Russian stock market index has a more dominant role in this interaction. Dedi & Yavas (2016) and Kocaarslan et al. (2017) revealed a volatility spillover effect from Türkiye to Russia. Berberoglu (2020) indicated ARCH and GARCH effects between the Turkish and Russian stock markets. Bayramoğlu & Abasız (2017) and Kutlu & Karakaya (2020) did not find any volatility relationship between indices in the post-crisis period. Additional volatility spillover studies between stock market indices considering Türkiye or Russia have been conducted, including Saleem (2009), Beirne et al. (2010), Gürsoy & Eroglu (2016), Celik et al. (2018), McIver & Kang (2020), and Mensi et al. (2020). Saleem (2009) found return and volatility linkages between Russia–US, Russia–European Union, Russia–Emerging Europe, and Russia–Asia, indicating that the relations are weak. Beirne et al. (2010) determined that there is a volatility spillover from global and local stock markets to Turkish and Russian stock indices. Gürsoy & Eroglu (2016) analyzed stock market transmissions among Türkiye, Brazil, India, Indonesia, and South Africa using VAR-EGARCH and did not find any relations from Brazil, India, Indonesia, and South Africa to Türkiye. Celik et al. (2018) examined return and volatility linkages among Islamic stock indices of the US, Indonesia, Malaysia, and Türkiye, demonstrating bidirectional volatility spillover between Indonesia and Türkiye, and a unidirectional volatility spillover from Türkiye to Malaysia.

In this study, univariate short and long memory ARCH models are first used. Considering the combined short and the long memory models in volatility modeling establishes a variation from similar previous studies. In addition, considering the Fourier series in volatility modeling (adaptive models) presents another a contribution to the literature. The findings indicate that the same ARCH model was the fittest method for both stock indices. The rolling Hong causality model is then used to capture the time-varying pairwise relationship (spillover, transmission) between the Turkish and Russian stock markets. Using the rolling Hong causality model allows us to clearly illustrate pre- and post-crisis effects. The analysis reveals a bidirectional volatility spillover between BIST 100 and RTSI throughout the analysis period.

These results are instructive for investors and policymakers because investors follow stock market volatility to optimize portfolio structure and avoid risk. Policymakers focus on volatility to avoid the spillover effects from significant changes in financial markets (Wang et al., 2020) because volatility is assumed to be an indicator of the vulnerability or stability of financial markets and the economy (Yu, 2002; Poon & Granger, 2003). This section has provided a brief summary of the theories and literature relating to volatility modeling. The next section describes the methodologies to determine the internal and external volatility structures.
Methodology

Short and long memory ARCH models and the rolling Hong causality model are used in this study. For these reasons, the methodology section was examined under two subheadings.

Autoregressive Conditional Heteroskedasticity Models

Bollerslev (1986) introduced the GARCH model as an extension of ARCH models similar to the AR portion of the autoregressive moving average (ARMA) model. The formulation of the GARCH model is as follows:

\[ h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i e_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \]  

(1)

\[ p \geq q > 0 \]

\[ \alpha_0 > 0 \quad \alpha_i \geq 0 \quad i=1, \ldots, q \]

\[ \beta_i \geq 0 \quad i=1, \ldots, p \]

In the GARCH model, conditional variance and lagged conditional variance are linear functions of a past sample. The effect of shocks on volatility and the persistence of volatility clustering are demonstrated by \( \alpha \) and \( \beta \), respectively (Yildirim et al., 2020).

Nelson (1991) demonstrated that bad and good news exert different effects on volatility and can be captured by the EGARCH model. If bad news has a greater impact on volatility, the scenario is referred to as the leverage effect. The EGARCH model can be written as follows:

\[ \ln(\sigma_t^2) = \alpha_t + \sum_{k=1}^{m} \beta_k g(Z_{t-k}) \]  

(2)

\[ g(Z_t) \equiv \theta Z_t + \gamma [ |Z_t| - E |Z_t| ] \]  

(3)

where \( \gamma [ |Z_t| - E |Z_t| ] \) indicates the magnitude effect of GARCH. If the magnitude of \( Z_t \) is higher (lower) than the expected value, \( \ln(\sigma_{t+1}^2) \), the process becomes positive (negative). If returns become negative (positive), the conditional variance is positive (negative) when \( \gamma = 0 \) and \( \theta < 0 \).

Another asymmetric model demonstrating the effect of shocks on volatility is GJR-GARCH which was introduced by Glosten, Jagannathan, & Runkle (1993). The equation can be formulated as follows:
When \( \varepsilon_{t-j} < 0 \) and \( \varepsilon_{t-j} \geq 0 \), the dummy variable becomes 1 and 0, respectively (Poon, 2005). To detect the leverage effect, \( \delta_j \) should be higher than 0 (Brooks, 2008).

The APARCH model was suggested by Ding et al. (1993), which allows for capturing the leverage effect on volatility similar to EGARCH and GJR-GARCH. It can be written as follows:

\[
S_t^\delta + \sum_{i=1}^{p} \alpha_i \left( |\varepsilon_{t-i}| - Y_i \varepsilon_{t-i} \right)^\delta + \sum_{j=1}^{q} \beta_j S_{t-i}^\delta
\]

\( \alpha_0 > 0 \quad \delta \geq 0 \)
\( \alpha_i \geq 0 \quad i=1, \ldots, p \)
\( -1 < Y_i < 1 \quad i=1, \ldots, p \)
\( \beta_j \geq 0 \quad j=1, \ldots, q \)

The APARCH model implements the Box–Cox power transformation of conditional standard deviation and asymmetric absolute residuals.

Baillie et al. (1996) introduced the fractional integrated GARCH (FIGARCH) model using GARCH and IGARCH models. The FIGARCH model indicates the long memory property of volatility. FIGARCH formulations are as follows:

\[
[1 - \beta(L)]\sigma_t^2 = \omega + [1 - \beta(L) - \varphi(L)(1 - L)^d]\varepsilon_t^2
\]

\[
\sigma^2 = \omega [1 - \beta(1)]^{-1} + \lambda(L)\varepsilon_t^2
\]

In Equation (6), \( d \) must be between 0 and 1. A hyperbolic (slow) decrease in volatility in the FIGARCH model is a more realistic approach than the exponential (fast) decrease in the GARCH model and the infinite persistence in the IGARCH model (Baillie et al., 1996).

The Baillie, Bollerslev, & Mikkelsen (BBM) and Chung approaches were used to estimate the parameters of the FIGARCH method. Chung asserted that the FIGARCH–BBM model had some specification and underscore problems, suggesting that the fractional differencing operator should be applied to the constant term in the mean equation (ARFIMA), but that no such approach is found in the variance equation (Chang et al., 2012; Al-Hajieh, 2017).

Bollerslev & Mikkelsen (1996) suggested a new method to capture the long memory and leverage effects of volatility simultaneously, which is referred to as the FIEGARCH model. The FIEGARCH equation can be written as follows:
\[
\ln(\sigma_t^2) = \omega + \varphi(L)^{-1}(1-L)^{-d}[1 + \psi(L)]g(Z_{t-1})
\]  

There is no nonnegativity constraint in the FIEGARCH model, as with EGARCH, and the \(d\) parameter must be between 0 and 1.

The FIAPARCH model was first introduced by Tse (1998) in an examination of the yen/dollar exchange rate, using the APARCH method using fractional integration. The equation processes for the relevant method are as follows:

\[
\sigma_t^\delta = \omega + \lambda(L) (|\varepsilon_t| - \gamma \varepsilon_t)^\delta
\]
\[
\omega = \eta/(1 - \beta)
\]
\[
\lambda(L) = \sum_{i=1}^{\infty} \lambda_i L = 1 - (1 - \beta L)^{-1} (1 - \phi L)(1 - L)^d
\]

where \(0 < d < 1\) determines that the effect of a shock on volatility is long memory. In equation (9), the expressions must be \(|\gamma| < 1\) and \(\delta > 0\). When \(\gamma > 0\) (\(\gamma < 0\)), this indicates that negative (positive) shocks increase volatility more than positive (negative) shocks.

Davidson (2004) introduced the HYGARCH model to investigate only the long memory properties of volatility, the formulations of which are as follows:

\[
\theta(L) = 1 \delta L / \beta L (1 + \alpha(1 - L)^d) - 1
\]

When \(\alpha = 1\) and \(\alpha = 0\), the HYGARCH model becomes FIGARCH and stable GARCH, respectively.

**Rolling Hong Causality Model**

This model was introduced to the literature by Hong (2001), and is calculated by considering a rolling correlation in the causality relationship method in mean and variance. To calculate the model, the standardized residuals and the squared standardized residuals obtained from the appropriate ARCH models must be determined. The equation processes of the rolling Hong model are as follows (Lu et al., 2014):

\[
r_{12,t}(J, S) = \frac{C_{12,t}(J, S)}{\sqrt{C_{11,t}(0, S)C_{22,t}(0, S)}}
\]
Equation (13) reveals the rolling correlation value in lag and indicate the subsample variances of and and identifies the cross-covariance between and .

\[
C_{12,t}(J, S) = \begin{cases} 
\frac{\sum_{i=0}^{S-j-1} u_{1,t-i} u_{2,t-i-j}}{S}, & j = 0, 1, \ldots, S - 1 \\
\frac{\sum_{i=0}^{S-j-1} u_{1,t-i} u_{2,t-i}}{S}, & j = -1, -2, \ldots, 1 - S 
\end{cases}
\]

\[
H_{1,t}^2(S) = \frac{S \sum_{j=1}^{S-1} k^2 \left( \frac{1}{M} \right) r_{12,t}^2(J, S) - C_{12}(k)}{2D_{12}(k)}
\]

Dataset

Türkiye and Russia are considered emerging economies and share an intense pairwise political, commercial, and financial relationship. Therefore, the aim of this study is to analyze the financial time series structures of the countries’ stock markets. To do so, two research questions are posed. First, which volatility models are the most appropriate for the stock market indices of these countries? Second, what is the direction of the volatility transmission mechanism between the stock market indices? The BIST 100 and RTSI indices are considered the leading indicators for stock markets, and this study uses 4,564 pieces of daily data spanning the years 2003–2020 for the investigation. The datasets are obtained from the Bloomberg terminal database. The price series graphs of the BIST 100 and RTSI indices are presented in Figure 1.

Figure 1 indicates that the BIST 100 had an increasing trend in the 2003–2020 period, while the RTSI increased significantly from 2003 to 2008. In addition, the figure demonstrates that 2008 global financial crisis clearly had an extremely significant effect on the RTSI.
The graphics of the return series obtained from the formula $\ln \left( \frac{P_t}{P_{t-1}} \right) \times 100$ are presented in Figure 2.

![Figure 2. Return Series Graphs of BIST 100 and RTSI](image)

According to Figure 2, the BIST 100 data demonstrate a more fluctuating structure compared to RTSI during the 2003–2020 period. The RTSI return series indicates that remarkable fluctuations were experienced in 2008, and the data are gathered around zero. This presents knowledge about the previous stationarity of the return series. Descriptive statistics for the return series are presented in Table I.

<table>
<thead>
<tr>
<th></th>
<th>BIST100</th>
<th>RTSI</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.051037</td>
<td>0.026035</td>
</tr>
<tr>
<td><strong>Maximum</strong></td>
<td>12.12810</td>
<td>20.20392</td>
</tr>
<tr>
<td><strong>Minimum</strong></td>
<td>-13.33586</td>
<td>-21.19942</td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
<td>1.671287</td>
<td>2.029094</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>-0.354043</td>
<td>-0.572411</td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td>8.142499</td>
<td>14.93373</td>
</tr>
<tr>
<td><strong>JB</strong></td>
<td>5180.500***</td>
<td>27631.02***</td>
</tr>
<tr>
<td><strong>ADF (C+T)</strong></td>
<td>-19.81860***</td>
<td>-11.14133***</td>
</tr>
<tr>
<td><strong>PP (C+T)</strong></td>
<td>-67.49698***</td>
<td>-62.12004***</td>
</tr>
<tr>
<td><strong>KPSS (C+T)</strong></td>
<td>0.046271***</td>
<td>0.082604***</td>
</tr>
<tr>
<td><strong>BIST 100</strong></td>
<td>1</td>
<td>0.417571</td>
</tr>
<tr>
<td><strong>RTSI</strong></td>
<td>0.417571</td>
<td>1</td>
</tr>
<tr>
<td><strong>ARCH–LM 1-2</strong></td>
<td>83.418***</td>
<td>200.95***</td>
</tr>
<tr>
<td><strong>ARCH–LM 1-5</strong></td>
<td>49.640***</td>
<td>120.25***</td>
</tr>
<tr>
<td><strong>ARMA</strong></td>
<td>(0,0)</td>
<td>(1,1)</td>
</tr>
</tbody>
</table>

Note: The expressions ***, **, and * indicate significance at 99%, 95%, and 90% confidence intervals, respectively.

Table I demonstrates that the mean value in both stock markets is positive; however, the value is higher in the BIST 100. The standard deviation values that indicate the deviation of a series from the mean (i.e., risk) are higher in the RTSI. The fact that the skewness values differ from 0 and are negative indicates that the series is skewed to the left, implying that the
probability of the occurrence of negative events is higher than that of positive events. A kurtosis value greater than three indicates that the dataset is leptokurtic, meaning that the dataset is distributed around zero. The Jarque–Bera (JB) values indicate that the dataset is normally distributed. The stationary structure of the series is tested using ADF, PP, and KPSS. ADF and PP tests examine the null hypothesis that the series has a unit root, and KPSS analyzes the null hypothesis that the series is stationary. According to the results in Table I, ADF and PP tests reject the null hypothesis, and the KPSS does not reject stationarity. In addition, Table I reveals a positive correlation of 0.41 between the BIST 100 and the RTSI. The ARCH–LM tests in Table I indicate a problem of heteroskedasticity in both datasets at the second and fifth lags.

Empirical Results

Three different classifications (short memory, long memory, and asymmetric) are used to determine the most appropriate volatility structure for the BIST 100 and RTSI indices. The short memory models indicate that the correlation in the dataset exists at short lags and the shock in the dataset is mean reversion in the short-term. The long memory models imply that the dataset has autocorrelation at high lags and the shock presents mean reversion in the long run. Asymmetric models indicate that the effect of negative or positive shocks on volatility differ.

Tables II and III present the comparison of the volatility models in the form of short memory, long memory, and asymmetric structure.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>BIST 100 Model Comparison Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GARCH</td>
</tr>
<tr>
<td>LL</td>
<td>−8290.387</td>
</tr>
<tr>
<td>AIC</td>
<td>3.635139</td>
</tr>
<tr>
<td>SIC</td>
<td>3.642179</td>
</tr>
<tr>
<td>FIGARCH-CHUNG</td>
<td>FIEGARCH</td>
</tr>
<tr>
<td>LL</td>
<td>−8280.610</td>
</tr>
<tr>
<td>AIC</td>
<td>3.631293</td>
</tr>
<tr>
<td>SIC</td>
<td>3.639741</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3</th>
<th>RTSI Model Comparison Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GARCH</td>
</tr>
<tr>
<td>LL</td>
<td>−8613.640</td>
</tr>
<tr>
<td>AIC</td>
<td>3.777669</td>
</tr>
<tr>
<td>SIC</td>
<td>3.787524</td>
</tr>
<tr>
<td>FIGARCH-CHUNG</td>
<td>FIEGARCH</td>
</tr>
<tr>
<td>LL</td>
<td>−8608.680</td>
</tr>
<tr>
<td>AIC</td>
<td>3.775933</td>
</tr>
<tr>
<td>SIC</td>
<td>3.787197</td>
</tr>
</tbody>
</table>
The model with the largest LL value and the smallest AIC and SIC values is determined to be the most appropriate. Tables II and III indicate that the FIAPARCH–CHUNG and FIAPARCH–BBM models are suitable for BIST 100 and RTSI, respectively.

The next section of the study compares FIAPARCH, ICSS–FIAPARCH, and adaptive FIAPARCH models for both datasets. To construct the ICSS model, the breaks in variance are first determined using the KAPPA-2 method to capture variance breaking dates because of the consideration of non-mesokurtic structure and persistence in conditional variance (Çağlı et al., 2011). Graphs of variance breaks determined applying KAPPA-2 are presented in Figure 3.

![Figure 3. Breaks in variance in return series](image)

*Note: Boundary lines are ± 3 standard errors*

Figure 3 reveals six variance breaking points in the BIST 100 and two in the RTSI. The 2008 global financial crisis clearly affected variance breaking points in both datasets. These break dates are used to determine the ICSS–FIAPARCH model.

The Fourier series is added to construct the adaptive FIAPARCH model. The A-FIGARCH model proposed by Baillie and Morana (2009) analyzes the case of structural breaks or regime changes in the constant term; that is, in the unconditional variance. Hence, the authors applied Gallant’s (1984) smooth flexible functional form. The Fourier formulation is as follows:
The graphic form of the Fourier series is illustrated in Figure 4.

![Figure 4. Fourier series](image)

The most appropriate base model, the ICSS model, and the adaptive model results determined for BIST 100 and RTSI indices, are presented in Table IV.

<table>
<thead>
<tr>
<th></th>
<th>BIST100</th>
<th></th>
<th></th>
<th>RTSI</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MODEL 1</td>
<td>MODEL 2</td>
<td>MODEL 3</td>
<td>MODEL 1</td>
<td>MODEL 2</td>
<td>MODEL 3</td>
</tr>
<tr>
<td>(\omega)</td>
<td>4.224421***</td>
<td>3.009763***</td>
<td>3.137936***</td>
<td>0.112029***</td>
<td>0.126142***</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>[5.690]</td>
<td>[6.083]</td>
<td>[2.989]</td>
<td>[3.405]</td>
<td>[5.663]</td>
<td>-</td>
</tr>
<tr>
<td>(d)</td>
<td>0.307122***</td>
<td>0.211608***</td>
<td>0.241885***</td>
<td>0.405672***</td>
<td>0.409641***</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>[8.852]</td>
<td>[6.776]</td>
<td>[5.967]</td>
<td>[5.663]</td>
<td>[5.663]</td>
<td>-</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.186002**</td>
<td>0.135733</td>
<td>0.174595**</td>
<td>0.200333***</td>
<td>0.201357***</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>[2.319]</td>
<td>[1.989]</td>
<td>[4.080]</td>
<td>[4.106]</td>
<td>[4.106]</td>
<td>-</td>
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<tr>
<td>(\beta)</td>
<td>0.412952***</td>
<td>0.277832***</td>
<td>0.343133***</td>
<td>0.533540***</td>
<td>0.536624***</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>[4.638]</td>
<td>[3.590]</td>
<td>[6.800]</td>
<td>[6.623]</td>
<td>[6.623]</td>
<td>-</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0.649391***</td>
<td>0.822210***</td>
<td>0.815943***</td>
<td>0.319848***</td>
<td>0.340128***</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>[4.112]</td>
<td>[3.800]</td>
<td>[4.545]</td>
<td>[4.564]</td>
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<tr>
<td>(\delta)</td>
<td>1.170064***</td>
<td>1.093716***</td>
<td>1.119134***</td>
<td>1.724808***</td>
<td>1.658537***</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>[7.192]</td>
<td>[6.627]</td>
<td>[14.84]</td>
<td>[13.29]</td>
<td>[13.29]</td>
<td>-</td>
</tr>
<tr>
<td>Student</td>
<td>5.945757***</td>
<td>6.084311***</td>
<td>6.077801***</td>
<td>5.381339***</td>
<td>5.345895***</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>[10.94]</td>
<td>[10.67]</td>
<td>[12.11]</td>
<td>[12.05]</td>
<td>[12.05]</td>
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</tr>
<tr>
<td>Q(50)</td>
<td>60.3676</td>
<td>59.6867</td>
<td>57.5947</td>
<td>43.8765</td>
<td>43.5373</td>
<td>-</td>
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<tr>
<td>Q(50)²</td>
<td>52.7277</td>
<td>66.9537</td>
<td>61.4642</td>
<td>28.6420</td>
<td>28.9037</td>
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<tr>
<td>ARCH-1-5</td>
<td>0.21638</td>
<td>0.23296</td>
<td>0.33231</td>
<td>0.20671</td>
<td>0.18699</td>
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<tr>
<td>MSE</td>
<td>11.02</td>
<td>10.94</td>
<td>9.866</td>
<td>12.41</td>
<td>11.95</td>
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</tr>
<tr>
<td>MAE</td>
<td>2.127</td>
<td>2.12</td>
<td>2.132</td>
<td>3.463</td>
<td>3.39</td>
<td>-</td>
</tr>
<tr>
<td>TIC</td>
<td>0.6484</td>
<td>0.6426</td>
<td>0.5692</td>
<td>0.5132</td>
<td>0.5098</td>
<td>-</td>
</tr>
<tr>
<td>LL</td>
<td>-8246.242</td>
<td>-8238.454</td>
<td>-8239.773</td>
<td>-8589.664</td>
<td>-8587.873</td>
<td>-</td>
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<tr>
<td>AIC</td>
<td>3.617109</td>
<td>3.615011</td>
<td>3.615150</td>
<td>3.768477</td>
<td>3.768130</td>
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<tr>
<td>SIC</td>
<td>3.628372</td>
<td>3.630498</td>
<td>3.629230</td>
<td>3.782556</td>
<td>3.783618</td>
<td>-</td>
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</table>

*Note:* The expressions ***, **, * show significance at 99%, 95%, and 90% confidence intervals, respectively.
The Model 1 (FIAPARCH) results in Table IV indicate that the d (long memory) parameter has a value between 0 and 0.5 in the BIST 100 and RTSI indices. This result indicates that both datasets have long memory, in which volatility clustering (autocorrelation) is long-term, revealing a hyperbolic mean reversion.

The $\gamma$ and $\delta$ coefficients ($-1 < \gamma < 1$ ve $0 < \delta < 2$) in Model 1 demonstrate a leverage effect for both datasets; however, the power parameter ($\delta$) of RTSI is larger. The leverage effect indicates that negative shocks have a stronger impact on volatility than positive shocks. The Model 2 (ICSS–FIAPARCH) results in Table IV indicate that parameter d decreases only in the BIST 100 compared with Model 1. This result implies that a variance break only causes spurious memory in the BIST 100. The Model 3 (adaptive FIAPARCH) results in Table IV show that the RTSI results are insignificant.

Two different methods are employed to determine which model better represents the dataset. The first of these is model comparison values, and the second is in the sample forecasting values. The model comparison values for the BIST 100 (LL, AIC, and SIC) show that the ICSS–FIAPARCH–CHUNG model is more appropriate; however, forecasting values (the small value is most suitable) show that the adaptive FIAPARCH–CHUNG method is more suitable. For the RTSI, both the model comparison and in-sample forecasting values indicate that the ICSS–FIAPARCH–BBM model is suitable.

Following the determination of appropriate volatility models for the BIST 100 and RTSI indices, the interaction between the datasets is analyzed. Accordingly, the time-varying Granger causality test, which was introduced to the literature by Lu et al. (2014), is used for mean and variance.

The variance causality approach uses the standardized residual values obtained from the GARCH method; therefore, model specifications are crucial to the power of the test (Yildirim et al., 2020). In addition, Van Dijk et al. (2005) and Rodrigues & Rubia (2007) argued that severe size distortion will occur if variance breaks are not considered. Therefore, the standardized residual values obtained from the ICSS–FIAPARCH models and the square of these values are used for the causality analysis to avoid size distortion and due to the suggestion of model selection criteria. Figures 5 and 6 present the rolling Hong causality in mean and variance.
Figure 5 reveals return spillover effects between the BIST 100 to the RTSI before the pre-crisis (2008 global financial crisis) and post-crisis period; however, during the crisis period, no return spillover is evident. A lack of return spillover during the crisis period is expected, because throughout that period, US stock markets had a dominating role in spillover. The relationship between Türkiye and Russia indicates that two important events in the last 10 years may have induced shock effects between the stock markets. These events included the shooting down of a Russian warplane by the Turkish Armed Forces on November 24, 2015 due to a border violation, and the assassination of the Turkish Russian Ambassador Andrey Karlov on December 19, 2016. Figure 5 indicates that no shock transfer is detected between the markets on either date. The most striking finding in Figure 5 is a return spillover from the RTSI to the BIST 100 only occurred during the COVID-19 period. This result indicates a shock transmission from the RTSI to the BIST 100 that may be considered a “contagion effect.” Kutlu & Karakaya (2020) found return spillover from BIST to RTSI prior to the pre-crisis period and from RTSI to BIST in the crisis period, and they did not find any spillover in the post-crisis period.

The existence of strong causality in the return series may cause deceptive results in variance analysis; therefore, causality in the mean should be filtered out (Panatelidis & Pittis, 2004). To do so, lags of both stock market indices should be added to ARCH models to consider the direction of causality to obtain new squared standardized residuals (Yildirim et al., 2020).
Figure 6 reveals a bidirectional trajectory and risk spillover in all time horizons between the RTSI and the BIST 100. This result corroborates Yarovaya et al. (2016), Gökbulut (2017), and Berberoglu (2020) therefore, both indices do not represent risk diversification instruments for one another.

**Conclusion and Discussion**

Previous research has not thoroughly considered the internal and external characteristics of volatility at the same time. This study aims to fill this deficiency by considering the new methods of the adaptive volatility and rolling Hong causality models. To do this, BIST 100 and RTSI indices are selected as a sample.

ICSS–FIAPARCH–CHUNG and the adaptive FIAPARCH–CHUNG are determined to be the most appropriate models for the BIST 100 in terms of model selection criteria and in-sample forecasting values, respectively. For the RTSI, the ICSS–FIAPARCH–BBM model is suitable in terms of model selection criteria and in-sample forecasting values. These results demonstrate long memory and leverage effects in both indices. The long memory effect indicates that the volatility clustering (autocorrelation) is long-term, and this situation is a hyperbolic mean reversion. The long memory effect indicates a slow and long-term response to new information. This corroborates the rejection of the weak efficient market hypothesis because an efficient market instantly reacts to new information, implying that investors can use past price movements to estimate future price movements to obtain higher than average returns by applying technical analysis. The leverage effect indicates that negative news has a greater impact on volatility than positive news. It can be asserted that vulnerability is higher in the RTSI because the variance breaking dates only cause spurious memory in the BIST 100, and persistence and leverage effects were more dominant in the RTSI. The rationale for
these greater persistence and leverage effects could be more aggregated arrival of information and investors’ higher sensitivity of the regarding the RTSI. The Granger causality in the mean reveals return spillovers in different directions at different times. For example, during the COVİD-19 period, a return spillover only occurred from the RTSI to the BIST 100. This suggests that when predictions of the future return series of the BIST 100 during the COVİD-19 period should include the past return series of the RTSI; however, the spillover effects are insufficient for investors and policymakers decision-making processes, as they also need access to the spillover relations in second moments called volatility (variance) spillover. In terms of variance causality, bidirectional risk and information spillover (transfer) are evident between both indices in all time horizons, indicating that both indices are not portfolio diversification instruments for one another at any time, and when examining the BIST 100 (RTSI), we should also consider RTSI (BIT100).

A general assessment of the study’s findings indicates that stock markets have long memory and leverage effect properties; therefore, it is more beneficial to examine the causes of both features in volatility modeling. Other inferences revealed from the study include the more fragile nature of RTSI and the significance of regional proximity for risk management strategies. The fragile structure of the RTSI demonstrates that the RTSI is riskier than the BIST 10; therefore, investors and policymakers should consider this. Regional proximity, which is one of the reasons for risk and information transfer, should be considered by investors and policymakers to examine issues such as the detection of contagion effects and strategy formulation.

The results obtained from the study provide practical information for researchers, policymakers, and investors. Future studies should incorporate wavelet transformations, copula (tail dependence), and value-at-risk-based risk spillover methods to enhance the volatility modeling.

**References**


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