

COMPARISON OF TAX REVENUE FORECASTING MODELS FOR TURKEY

Hamza ERDOĞDU¹

Recep YORULMAZ²

Abstract

The objective of this study is to compare performance of three forecasting tax revenue models for Turkey over the period of 2006: 01 to 2018: 12. Three different time series forecasting techniques such as Random Walk, SARIMA (Seasonal Autoregressive Integrated Moving Average) and BATS (Exponential Smoothing State Space Model with Box-Cox Transformation, ARMA Errors, Trend and Seasonal Components) are used in the study. At the beginning of the analysis, the data set was apportioned into two parts: training and testing. The training period is from 2006: 01 to 2014: 12 and the testing part is from 2015: 01 to 2018: 12. Based on different evaluation criteria, forecast points of 36 months are obtained for each forecasting model. We find that using the BATS model, rather than classical S(ARIMA) in forecasting series of monthly tax revenues of Turkey, provide more accurate forecasts. The empirical findings of this study help the experts in the preparation process of government's budgets.

Keywords: Forecasting, Tax Revenue, BATS, SARIMA, Turkey

JEL Code: C1, C5, H20.

1. Introduction

Tax revenues are considered amongst the fundamental sources of governments budget planning. Governments collect taxes not only to finance their expenses but also aiming of stabilization, distribution and allocation in the economy. They use taxes to stabilize the employment levels, balance of payments and/or prizes. They might try to intervene the income and wealth distribution by playing with the tax structure. Further, they might want to use taxes to allocation of resources in the economy by using their allocative effects on certain goods (Brown & Jackson, 1986: 297).

There are three fundamental classifications in Turkish tax system. These are income taxes, taxes on expenditure and taxes on wealth. The relative importance of these taxes in the Turkish tax system is presented in Table 1. Income taxes are classified as individual income and corporate income taxes. Income taxes yield about 30% of total revenues in Turkish tax system.

¹ Asst. Prof. Dr., Harran University, Faculty of Economics and Administrative Sciences, Department of Econometrics, hamzaerdogdu@harran.edu.tr

² Asst. Prof. Dr., Ankara Yıldırım Beyazıt University, Faculty of Political Sciences, Department of Public Finance, ryorulmaz@ybu.edu.tr

Table 1. Percentage Distribution of Tax Revenues in Turkey

	2008	2009	2010	2011	2012
Income Taxes	31,3	31,8	28,4	29,1	29,7
Personal Income Taxes	21,2	20,9	18,4	18,5	19,2
Corporate Income Taxes	10,1	10,9	10	10,6	10,5
Taxes on Expenditure	66,5	65,4	69	68,6	67,9
Value Added Tax	32,5	31,7	33,2	34,7	33,6
Special Consumption Tax	22,7	23	25,4	23,3	23,4
Banking and Insurance					
Transaction Taxes	2	2,1	1,6	1,6	1,8
Stamp Duty Tax	2,2	2,2	2,3	2,4	2,4
Special Communication Tax	2,5	2,3	1,8	1,6	1,5
Tax on Wagering	0,2	0,2	0,2	0,2	0,2
Tax on Customs	1,5	1,3	1,4	1,7	1,7
Taxes on Wealth	2,3	2,5	2,3	2,3	2,3
Inheritance and Gift Taxes	0,1	0,1	0,1	0,1	0,1
Motor Vehicle Tax	2,2	2,4	2,2	2,2	2,2

Source: Saracoglu et. al. (2014)

Taxes on expenditures, on the other hand, contain approximately 68% of total revenues in Turkish tax system. Taxing expenditures is considered as the common and easy way to collect taxes for governments. Hence, that big amount of taxes is comprised of expenditures in Turkey. Finally, taxes on wealth only yield approximately 2% of total revenues. Tax analysis and forecasting of tax revenues for governments are crucial to ensure stability in tax and expenditure policies (Jenkins et. al., 2000).

Budgetary uncertainties directed governments to rely heavily on economic analysis in recent decades. Because of the extent of these fiscal problems forecasting tax revenues is essential for governments to manage their budget planning process. Recent fiscal problems of governments created reliability issues on economic and revenue forecasting. Hence, there are plenty of methods that are used to forecast tax revenues by policymakers (Fullerton, 1989). Transparency and accuracy are the key components while determining the method for forecasting. A potential manipulation of forecasts might create government problems.

Furthermore, inaccurate forecasts might hinder abilities of policymakers to make accurate budget planning and harm levels of productivity in the economy (Kyobe and Danninger, 2005; Cirincione et al., 1999). It is considered that countries with high-income levels and relatively small central government tend to have high formality, accuracy and transparency forecasts (Kyobe and Danninger, 2005).

Government revenue forecasting studies for Turkey are rare in the literature; hence this study aims to fill this gap. The rest of the study is organized as follows. Section II outlines the major studies that make forecasting analysis in the literature. Furthermore, Section III describes the methodologies of the forecasting techniques applied and the data that are used in the study. Section IV provides the outcomes of selected forecasting methods in the study. Finally, Section V contains the conclusion and discussions.

2. Literature

Majority of forecasting studies focused on the private sector in the literature so far, hence the studies focused on government revenue are relatively less than private sector studies. For instance, Gajewar and Bansal (2016) conducted forecasting analysis for private sector using machine-learning algorithms. Specifically, they performed ARIMA, ETS (Exponential Smoothing), STL (Seasonal and Trend Decomposition using Loess), and Random forest machine-learning algorithms to obtain revenue forecast for Microsoft. They suggested that using machine-learning algorithms methods would increase the accuracy of quarterly revenue forecasting.

Many researchers also focused on state and/or municipal revenue forecasting analysis so far. Fullerton (1989) analyzed sales tax revenues using composite forecasting model for Idaho. Using time series model and econometric models he examined the capability of composite forecasting model. He found that the composite forecast model are more effective than base line forecasts. The combined model was also found more accurate than previous forecast attempts for Idaho.

Hamboret. al. (1974) used econometric forecasting method using simple revenue structure for Hawaii. They forecasted state revenues including; excise, personal income, corporate income, and other state tax revenues, for a single fiscal year of Hawaii. Furthermore, Kyobe and Danninger (2005) analyzed the revenue forecasting practices in 34 low-income countries focusing especially on institutional prospects. They claimed that there are three key factors on forecasting practices such as “formality, organizational simplicity, and transparency”. They empirically found that countries levels of corruption are Assoc.d with formality and transparency of forecasting. Accordingly, they found that high levels of corruption are related with less formal and transparent forecasts.

Cirincione et al. (1999) examined the impact of using time series models, the length and the frequency of the data on non-tax general fund revenue forecasting for the municipalities of Connecticut. They found that exponential smoothing models are most effective on bimonthly data in which they claim local governments should rely on rather than monthly or quarterly data.

As we pointed above, there are plenty of methods that were used to forecast private sector or government/state/municipal revenues in the literature. It is also important to analyze the methods used in these studies. In case of the Box-Jenkins AutoRegressive Integrated Moving Average Model (ARIMA), researchers found different results in effectiveness of ARIMA model. For instance, Makridakis and Hibon (1995) claimed that the ARIMA model performs relatively poor than other models. In doing so, Makridakis et al. (1979) found the reason of this poor performance of ARIMA model as usage of differencing in order to find stationary in the mean of the series.

Similarly, in a series of studies that focused on local government revenue forecasting for the municipalities of Florida, researchers found similar results. They claimed that Box-Jenkins ARIMA model performs poorly than other methods such as time series models, which produce lower forecast errors. Furthermore, they found that trend fitting by regression generated more forecast errors than its counterpart methods (Frank and Gianakis, 1990; Gianakis and Frank, 1993).

It is important to point out that the studies that found poor performance for ARIMA method mainly focused on municipal government revenue forecasting. Differently, Downs and Rock (1983) found evidence that multivariate AutoRegressive Moving Average (ARMA) method is more effective than univariate techniques using ARMA model for municipal government revenue forecasting.

While, most of the forecasting studies examine relative performance of various methods so far, only few researchers tested the impact of data quality on the performance of forecasting methods. Gianakis and Frank (1993), which is one these studies, claimed that the length of the data does not have any impact on the accuracy of forecasting techniques. However, some scholars suggested that at least fifty observations are necessary to implement the Box-Jenkins ARIMA method. On the other hand, scholars have kept using this method with fewer numbers so far (Lorek et al., 1976).

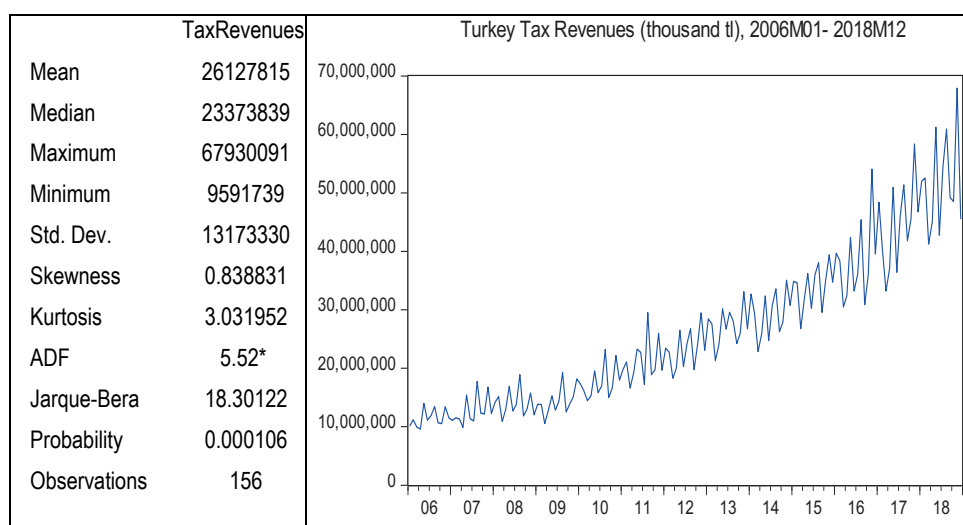
Lorek and McKeown (1978) analyzed the association between observation numbers and the performance of the Box-Jenkins method on quarterly market income data. They found that the forecast error is not significantly different in models based on fifty observations and models based on fewer observations. They suggested that if the number of observations of Box-Jenkins method decreases, forecast error increases. However, the performance of the model does not Assoc.d with the number of observations at least until twenty-four or fewer observations. Similarly, Lusk and Neves (1984) found a result consistent with the previous cases. They suggested that the performance of the Box-Jenkins model does not Assoc.d with the length of data or the frequency in their private sector study.

3. Data and Methodology

3.1. Data

In the analysis, the series of monthly tax revenues in the central government budget realisations, from January 2006 to December 2018 is used. The data is obtained from the web site of General Directorate of Budget and Fiscal Control (BÜMKO). The series is plotted and shown in Figure 1, provides also some descriptive statistics about the data to help better understanding the structure of the series. From the plot, it is clear that the series has trend and seasonal component.

Figure 1. Turkey Tax Revenues, 2006M01 – 2018M12



Source: General Directorate of Budget and Fiscal Control and General Directorate of Budget and Fiscal Control. * the t statistic value of the Augmented Dickey-Fuller test, indicating nonstationarity of the series at level 0.05.

3.2. Methodology

In this section, we provide the fundamentals of the forecasting methods used in the study such as: Random Walk, SARIMA and BATS.

3.2.1. Random Walk-RW

Random walk model is widely used in econometric forecasting studies as a benchmark.

A time series is said to follow a random walk process if the first differences are random.

For a time series Y_t , a *random walk* can be written as differences changes from one period to the next,

$$Y_t = Y_{t-1} + \varepsilon_t,$$

where Y_{t-1} is the value at time $t-1$ and ε_t is a discrete white noise at time t .

3.2.2. SARIMA

Introduced by Box and Jenkins (1970) ARIMA (Auto-Regressive Integrated Moving Average) models are a broad category of univariate models. In forecasting a time series, these models bring together three components: the auto-regressive (AR), the moving average (MA) part and the integrated (I) part. The AR part indicates that individual values in a variable of interest can be described by linear models based on its own lagged values. The MA part assumes that regression error is a linear combination error terms. The integrated part (I) shows the degree of differencing.

The ARMA (AutoRegressive, MovingAverage) model is defined as follows:

$$Y_t = \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \dots + \varphi_p Y_{t-p} + \alpha_t - \psi_1 \alpha_{t-1} - \psi_2 \alpha_{t-2} - \dots - \psi_q \alpha_{t-q} \quad (1)$$

where the Y_t 's are the original time series, the φ 's are the unknown autoregressive parameters, the ψ 's are the unknown moving average parameters and the α 's are the white noise error terms.

A modification for nonstationary series, known as ARIMA (p, d, q) is;

$$\varphi_p(B)(1-B)^d Y_t = \psi_q(B)\alpha_t \quad (2)$$

where B is the backshift operator, thus $BY_t = Y_{t-1}$ and $B^2 Y_t = Y_{t-2}$ and the d parameter indicates the order of differencing.

For seasonal series, a more general form of above equation, and known as the multiplicative seasonal ARIMA –SARIMA (p, d, q)(P, D, Q)_s process is given by;

$$\phi_p(B)\Phi_p(B)(1-B)^d(1-B^s)^D Y_t = \psi_q(B)\Theta_q(B^s)\alpha_t \quad (3)$$

where $(1 - B^s)Y_t = Y_t - Y_{t-s}$ and s is the number of seasons per year, $(1 - B^s)^d$, d and D are the orders of differencing. Also Φ_p , Φ_P , Ψ_q and Θ_Q are the polynomial functions of orders p, P, q and Q , respectively.

3.2.3. BATS

To model series having only one seasonal pattern, Winters (1960) introduces the standard Holt-Winters method. However, Taylor(2003) extends the standard method to a double seasonal Holt-Winters method, following mainly De Livera et. al (2012);

$$Y_t = L_{t-1} + B_{t-1} + S_t^{(1)} + S_t^{(2)} + D_t, \quad (1A)$$

$$L_t = L_{t-1} + B_{t-1} + \alpha D_t, \quad (1B)$$

$$B_t = B_{t-1} + \beta D_t, \quad (1C)$$

$$S_t^{(1)} = S_{t-k_1}^{(1)} + \lambda_1 D_t, \quad (1D)$$

$$S_t^{(2)} = S_{t-k_2}^{(1)} + \lambda_2 D_t, \quad (1E)$$

where L_t represents the level component of the series Y_t at time t ,

B_t represents the trend component of the series Y_t at time t ,

$S_t^{(i)}$ represents the i th seasonal component at time t ,

k_1 and k_2 are the periods of the seasonal cycles,

D_t is the disturbance (or prediction error),

α, β, λ_1 and λ_2 are the smoothing parameters,

Proposed by De Livera et. al (2012), the BATS models (Exponential Smoothing State Space Model with Box-Cox Transformation, ARMA Errors, Trend and Seasonal Components) are designed in the exponential smoothing framework. The models are constructed to handle more than one seasonality as well as complex seasonalities for example, non-nested, non-integer and large period seasonality. De Livera et. al (2012) extends the double seasonal Holt-Winters method by adding a Box-Cox transformation, ARMA errors, and T seasonal patterns:

$$Y_t^{(w)} = \begin{cases} \frac{Y_t^w - 1}{w}, & w \neq 0, \\ \log Y_t, & w = 0, \end{cases} \quad (2A)$$

$$Y_t^{(w)} = L_{t-1} + \phi B_{t-1} + \sum_{i=1}^T S_{t-k_i}^{(i)} + D_t, \quad (2B)$$

$$L_t = L_{t-1} + \phi B_{t-1} + \alpha D_t, \quad (2C)$$

$$B_t = (1 - \phi)B + \phi B_{t-1} + \beta D_t, \quad (2D)$$

$$S_t^{(i)} = S_{t-k_i}^{(i)} + \lambda_i D_t, \tag{2E}$$

$$D_t = \sum_{i=1}^p \psi_i D_{t-1} + \sum_{i=1}^q \zeta_i \eta_{t-i} + \eta_t, \tag{2F}$$

where; L_t represents the local level in period t ,

B_t represents the short-run trend in period t ,

B represents the long-run trend in period t ,

φ represents the damping parameter,

$S_t^{(i)}$ represents the i th seasonal component at time t ,

k_1, k_2, \dots, k_T are the seasonal periods,

D_t is an ARMA(p, q) process,

η_t is a Gaussian process with zero mean and constant variance σ^2 ,

α, β, λ_1 and λ_2 are the smoothing parameters,

4. Analysis and Empirical Results

At the beginning of the analysis, we split the data into a training and testing set. The training set covers the period from 2006:01 to 2014:12 and the testing part covers the period from 2015:01 to 2018:12. The training data set is used to only to estimate unknown model parameters. Once the model coefficients are estimated, forecasts for each model are made for the testing part. To evaluate forecast accuracy of each model, the testing data is used.

The results of the best ARIMA(0,1,2)(0,1,1)¹² model for the tax revenues series is given in the following Table 2. Automatic ARIMA selection option was used in the forecast package in R, the details can be found in Hyndman and Khandahar (2008).

Table 2. The results of the ARIMA (0,1,2)(0,1,1)¹² model

	Lagged length	Coefficients	Standard Error
MA	1	-1.0682	0.0963
MA	2	0.3932	0.0969
SMA		-0.4693	0.0908

The results of the BATS(0.377, {0,0}, 1, {12}) model for the series is provided in Table 3. The forecast package in R is used to get the results.

Table 3. The results of the BATS (0.377, {0,0}, 1, {12}) Model

Parameters	Coefficients
Lambda	0.376905
Alpha	0.2432854
Beta	0.01566971
Damping	1
Gamma Values	-0.1082185

After fitting three time series models: random walk, SARIMA and BATS, forecasts for the testing period, 2015: 01 to 2018: 12, are obtained in Table 4.

Finally, accuracy of each model is measured on the testing set. Table 5, provides statistical measures of accuracy of each method based on various forecast evaluation criteria: ME, RMSE, MAE, MPE, MAPE, MASE and Theil's U.

Table 4. Point Forecasts of the Methods for the Testing Data (2016M01-2018M12)

Forecast Horizon	ACTUAL	Forecasting Methods		
		Random Walk	SARIMA	BATS
Jan 2016	39685212	34729587	39551109	38989627
Feb 2016	38361380	34729587	38414335	37396828
Mar 2016	30496694	34729587	31463837	30878845
Apr 2016	32446011	34729587	35440194	34802877
May 2016	42368600	34729587	40607697	41212156
Jun 2016	33195345	34729587	34699758	35611013
Jul 2016	36111701	34729587	39678120	38694089
Aug 2016	45425215	34729587	41889999	43894982
Sep 2016	30883849	34729587	34262776	34663442
Oct 2016	36060795	34729587	38123593	38056660
Nov 2016	54060129	34729587	43576351	44574433
Dec 2016	39906810	34729587	38712079	38555889
Jan 2017	48420673	34729587	43405126	43683557
Feb 2017	39994384	34729587	42309290	41972917
Mar 2017	33201256	34729587	35358791	34951174
Apr 2017	37082457	34729587	39335148	39182860
May 2017	50949456	34729587	44502651	46067487
Jun 2017	36422643	34729587	38594712	40052674
Jul 2017	46062984	34729587	43573075	43366295
Aug 2017	51377479	34729587	45784953	48940777
Sep 2017	41837993	34729587	38157730	39032728
Oct 2017	45559415	34729587	42018548	42681794
Nov 2017	58372034	34729587	47471306	49667752

Dec 2017	46766884	34729587	42607034	43217915
Jan 2018	51995609	34729587	47300081	48714507
Feb 2018	52558220	34729587	46204244	46882727
Mar 2018	41249512	34729587	39253745	39342516
Apr 2018	45049034	34729587	43230102	43890960
May 2018	61218542	34729587	48397605	51264277
Jun 2018	42749559	34729587	42489666	44824230
Jul 2018	54360053	34729587	47468029	48374916
Aug 2018	60934207	34729587	49679907	54333154
Sep 2018	49235735	34729587	42052684	43729821
Oct 2018	48504135	34729587	45913502	47642028
Nov 2018	67930091	34729587	51366260	55108914
Dec 2018	45525901	34729587	46501988	48216072

Table 5. Measures Accuracy of the Methods for Testing Set (2016M01-2018M12)

<i>Methods</i>	ME	RMSE	MAE	MPE	MAPE	MASE	Theil's U
Random Walk	10159746.6	13580773	10905568	19.50	21.87	3.93	1.27
SARIMA	2961666.3	5761357	4317229	4.84	8.73	1.56	0.55
BATS	2042864.3	4578144	3583768	3.13	7.43	1.29	0.44

The measures calculated are: *ME*: Mean Error, *RMSE*: Root Mean Squared Error, *MAE*: Mean Absolute Error, *MPE*: Mean Percentage Error, *MAPE*: Mean Absolute Percentage Error, *MASE*: Mean Absolute Scaled Error, *Theil's U*: Theil Inequality Coefficient.

Among the three generated models, the accuracies of the models are tested based on seven forecast evaluation criteria. From the Table 5 results, BATS is preferred as the best forecasting model for the tax revenues series of Turkey, since it provides lesser values of seven evaluation criteria: ME, RMSE, MAE, MPE, MAPE, MASE and Theil's U.

5. Conclusion

The aim of this study is to evaluate performance of three forecasting tax revenue models for Turkey over the period of 2006: 01 to 2018: 12. Three different time series forecasting techniques such as Random Walk, SARIMA (Seasonal Autoregressive Integrated Moving Average) and BATS (Exponential Smoothing State Space Model with Box-Cox Transformation, ARMA Errors, Trend and Seasonal Components) are used in the study. At the beginning of the analysis, the data set was apportioned into two parts: training and testing. The training period is from 2006: 01 to 2014: 12 and the testing part is from 2015:01 to 2018:12. Based on different evaluation criteria, forecast points of 36 months are obtained for each forecasting model.

BATS model outperforms the benchmark RW and SARIMA models based on all evaluation criteria. We find that using the BATS model, rather than seasonal ARIMA Yilmaz (2019) in forecasting series of monthly tax revenues of Turkey, provide more accurate forecasts. The empirical findings of

this study help the experts in the preparation process of government's budgets. For further forecasting of other tax types such as Corporate Income Tax, Value Added Tax and Total Tax, the BATS model may provide better performance. Also the empirical results of the current study will be used to develop combined forecasting models.

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