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Classical MCDM Methods and Their Operators on Grey Numbers Through Operator Overloading

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Abstract

This study investigates the possibilities of applying classical MCDM methods to grey numbers by leveraging operator overloading and multiple dispatch features of Julia programming language. Grey versions of the classical MCDM methods differ in some of their computational steps. In this paper, it is shown that a set of MCDM methods, including TOPSIS, ARAS, WASPAS, and EDAS are directly applicable to grey numbers through operator overloading without changing their core algorithms. These processes are implemented by re-defining arithmetic and comparison operators for grey numbers. We also compared our methodology with two existing Grey TOPSIS methods. It is shown that the sorted grey scores can yield the same rankings as the existing methods in some cases. When the results are not the same, we show that an appropriate whitening parameter can be found to obtain the same rankings. This approach preserves the uncertainty from the very beginning to the end of the process and provides a clear distinction between objective computational steps and subjective ranking steps. Switching the whitening parameter from zero to one also provides a sensitivity analysis tool to investigate how uncertainty affects the final rankings

Keywords multi-criteria decision making · grey numbers · operator overloading

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Introduction

Decision making is the process of identifying and choosing alternatives based on the values and preferences of the decision-maker (Simon, 1977). Decision-makers generally make decisions under uncertainty. The way this uncertainty is defined and its magnitude provides insight into the tools we will use to solve the decision problem.

The methods used to solve a decision problem vary according to many criteria. The most fundamental distinction is between intuitive and rational decisions (Tversky and Kahneman, 1974; Kahneman and Tversky, 1984). Intuitive decisions have been studied for many years as a subject of psychology, using an approach that models the admirable structure of human intelligence in terms of both inputs and outputs. When it comes to numerical decisions, the methods used for the solution vary depending on the decision environment, number of decision-makers, and number of criteria.The solution for single-criteria decisions is relatively easy. There is also a consensus in the literature on this topic.However, when it comes to multi-criteria decision making, choosing the method to solve the decision problem becomes a decision problem in itself. In the literature, new methods are being developed every day and a rapidly growing body of knowledge is being formed.

Multi-criteria decision-making (MCDM) is the task of defining a special measure to rank or prioritize alternatives through a decision matrix that includes multiple predetermined criteria, weights assigned to these criteria, and optimization directions of the criteria. The fundamental problem here is defining a ranking in a multidimensional space by nature. In contrast, ranking is precisely defined over the set of real numbers. For example, when we look at the sequence (2, 5, 7), it can be called ordered because 2 ≤ 5 and 5 ≤ 7. However, if we define the problem as ordering the points (1, 3) and (5, 7), the situation is different from the previous example. In multidimensional spaces, there is no precise and generally accepted definition for ranking and prioritization operations based on binary comparison operators such as ≤, <, >, and ≥. In this case, the problem of ordering the points (1, 3) and (5, 7) cannot be achieved in a single way. This example can be generalized to n-dimensional space. As a result, in multi-criteria decision-making problems, an infinite number of rankings are possible.

There are many MCDM methods in the literature. These methods, known as classical MCDM methods, are preferred when criteria and alternatives are clearly defined and based on precise data. These methods rely on various fundamental principles for evaluating and ranking alternatives, such as distance (TOPSIS (Hwang andYoon, 1981), VIKOR (Yazdani and Graeml, 2014), EDAS (Keshavarz Ghorabaee et al., 2015), ratio (ARAS (Zavadskas and Turskis, 2010), WSM and WPM (Chourabi et al., 2019), WASPAS (Zavadskas et al., 2012), pairwise comparison (AHP (Saaty, 2008, 1980), ELECTRE (Roy, 1968)), and weight and preference functions (PROMETHEE (Brans et al., 1986)). Thanks to these principles, it is possible to select a method suitable for the decision maker and the nature of the decision problem.

In addition, the theoretical foundations, classification, and application areas of classical MCDM methods have been systematically discussed in the literature through comprehensive and comparative studies such as Triantaphyllou (2000), Zavadskas and Turskis (2011), and Mardani et al. (2015). Furthermore, innovative and hybrid MCDM approaches have been proposed in the literature to overcome the limitations of classical methods, particularly by combining ranking and classification perspectives. In this context, the MARCOS approach developed by Stević et al. (2020) is based on the evaluation and ranking of alternatives with respect to a compromise solution while the CoCoSo method developed by Yazdani et al. (2019b) presents a combined decision-making framework based on a combination of compromise approaches. However, there are also methods developed using an original combination of the steps of existing methods. Puška et al. (2022) proposed the CRADIS approach, in which the steps of the ARAS, TOPSIS, and MARCOS methods are used together to select the most appropriate incinerator for healthcare waste management. Baki (2023), on the other hand, proposed the CAPMA method, which compares the current market with potential markets and ranks potential market alternatives using classification coefficients calculated based on the utility degrees obtained from the entropy-based ARAS method.

In most classical MCDM methods, the elements of the decision matrix consist of quantitative or qualitative data, but during the steps of the methods, qualitative data are typically quantified to obtain a decision matrix composed entirely of real numbers. However, multi-criteria decision problems are inherently complex, and decision makers' evaluations are often subjective, and/or information about alternatives and criteria can be incomplete or uncertain. In the literature, Fuzzy MCDM and Grey MCDM methods based on theories such as Fuzzy Number Theory and Grey System Theory have been developed to address such situations. Fuzzy MCDM methods provide an effective solution for situations involving uncertainty and subjective evaluations, while Grey MCDM methods offer an effective solution for situations with incomplete or uncertain information. Recent years have witnessed a growing research interest in developing methodological frameworks that enable the systematic incorporation of incomplete, uncertain, and interval-valued information into decision-making processes, with the aim of ensuring reliable and consistent outcomes. In this context,Radovanović et al. (2025) proposed a robust ranking approach by embedding interval grey numbers within the Grey MARCOS method, allowing uncertain and partially known evaluations to be assessed through comparisons with ideal and anti-ideal solutions. Lin et al. (2025) represent uncertainty in decision-making problems using three-parameter interval grey numbers and propose an innovative decision-making framework that integrates a grey relational clustering approach with a genetic algorithm to dynamically optimize key parameters. Zhang and Esangbedo (2025) proposed a novel MCDM methodology grounded in Grey Systems Theory, aiming to reduce uncertainty and cognitive biases in expert judgments; by integrating System-2 (reflective) thinking with grey-based weighting and ranking techniques, they demonstrated that more consistent and strategically aligned outcomes can be achieved in both theoretical and practical decision-making problems. This study focuses only on MCDM methods based on Grey System Theory.

In multi-criteria decision-making problems, Grey MCDM methods offer a solution to the situation where the elements of the decision matrix consist of grey numbers, based on the principle of implementing the steps of classical MCDM methods using grey numbers. Some of the pioneering studies in this area include TOPSIS-G (Lin et al., 2008), ARAS-G (Turskis and Zavadskas, 2010), WASPAS-G (Zavadskas et al., 2015), and EDAS-G (Stanujkic et al., 2017) which have emerged as a broad research field in the literature. However, in Grey MCDM methods, there are some differences observed in the calculation steps. The analysis starts with a decision matrix containing grey numbers, and some studies differ in the normalization step, while others differ in which step to whiten the grey numbers and how to obtain the scores. Although these differences are understandable in terms of both the structure of the decision problem and the mathematical background, it is clear that there is a lack of standardization in the field.

The fundamental reason for this difference in Grey MCDM methods is the computational difficulties in operations involving grey numbers. Actually, it is not impossible to eliminate these differences. Grey System Theory clearly presents grey numbers and operations defined on grey numbers (Deng, 1982, 1985). Counterparts of operations used in the steps of classical MCDM methods are defined on grey numbers. These operations can be performed quickly and accurately through computer software. In this context, the

Julia (Bezanson et al., 2017) programming language can be used as an important tool in the application of Grey System Theory by providing high performance and flexibility. With Julia's multiple dispatch feature, arithmetic, logical, and comparison operations defined on grey numbers can be easily implemented within a user-friendly language structure. These features enable grey number operations to be performed more systematically and efficiently. It is possible to implement all steps of classical MCDM methods on a decision matrix composed of grey numbers without whitening the data at any stage. Implementations are free to download within a Julia package JMcDM (Satman et al., 2021).

In this context, the study aims to address the computational and standardization issues encountered in multi-criteria decision-making problems involving grey numbers by leveraging abstraction capabilities at the programming language level. Accordingly, this study seeks to answer the research questions "Can classical Multi-Criteria Decision-Making methods be directly applied to grey numbers while preserving their algorithmic structure by using Julia's operator overloading and multiple dispatch features?" and "Can a clear and consistent distinction be established between objective computational steps and subjective evaluation steps in the multi-criteria decision-making process through operations defined on grey numbers?" Unlike existing studies in the literature, this study considers all steps of the decision-making process with grey numbers without requiring whitening at any stage. In this respect, the study provides a methodological contribution to the field of multi-criteria decision-making with grey numbers by preserving algorithmic integrity and offering a generalizable approach. This study focuses on solving MCDM problems involving grey numbers, specifically within the context of the TOPSIS method. The fact that arithmetic operators are largely defined on grey numbers allows most algebraic operations to be objectively implemented using the concept of operator overloading. However, steps involving minimum and maximum operators require comparison operators, which remain subjective, essentially reflecting the very problem MCDM methods aim to address. Nevertheless, one of the existing methods for comparison in the literature can be selected, and the process can proceed similarly to classical TOPSIS. Although the study opens a discussion specific to TOPSIS, the points raised are also applicable to other methods such as EDAS (Keshavarz Ghorabaee et al., 2015), ARAS (Zavadskas and Turskis, 2010), and WASPAS (Zavadskas et al., 2012).

This study focuses on solving MCDM problems involving grey numbers, specifically within the context of the TOPSIS method. The fact that arithmetic operators are largely defined on grey numbers allows most algebraic operations to be objectively implemented using the concept of operator overloading. However, steps involving minimum and maximum operators require comparison operators, which remain subjective, essentially reflecting the very problem MCDM methods aim to address. Nevertheless, one of the existing methods for comparison in the literature can be selected, and the process can proceed similarly to classical TOPSIS. Although the study opens a discussion specific to TOPSIS, the points raised are also applicable to other methods such as EDAS (Keshavarz Ghorabaee et al., 2015), ARAS (Zavadskas and Turskis, 2010), and WASPAS (Zavadskas et al., 2012).

In Section 2, we review the existing works on Grey MCDM methods. In Section 3, we present the basic operators defined on grey numbers. In Section 4, we present the applicability of arithmetic and grey comparison operators and effects of whitening.We also discuss the possibilities of using the classical MCDM methods with grey numbers without changing the core algorithm thanks to operator overloading and the multiple dispatch. In Section 5, we present the experimental results of the study with two case studies using pseudo-grey and real-grey numbers. In this section, we show that the proposed methodology is applicable to grey numbers without changing the core algorithm of classical MCDM methods including TOPSIS, ARAS, WASPAS, and EDAS. We also discuss the effects of the whitening parameter on the result in a context of sensitivity analysis. We also compare our results with the existing studies that have examples applied using grey TOPSIS alternatives in the literature. Finally in Section 6, we conclude.

Related Works

The first research on grey systems titled Control Problems of Grey Systems by Professor Deng Julong of Huazhong University of Science and Technology in China was published in the journal Systems and Control Letters in 1982. According to Deng (1982), a grey system is defined as a system containing knowns and unknowns. The key concept behind Grey System Theory is the idea of grey information, which refers to information that is incomplete, uncertain, or partially known. There are four possibilities for incomplete information in systems, namely incomplete information related to elements (or parameters), structure, boundary conditions, and system behavior (Liu and Lin, 2006).

In real-world applications, the inability to express criteria and alternatives with precise numerical values makes uncertainty a natural part of the decision-making process. In this context, grey numbers provide an effective tool for representing partial information and uncertainty, and this need has led to the development of Grey MCDM approaches that integrate classical MCDM methods with grey numbers. In the literature, many grey versions of classical MCDM methods have been developed, such as TOPSIS-G (Lin et al., 2008), ARAS-G (Turskis and Zavadskas, 2010),WASPASG (Zavadskas et al., 2015), EDAS-G (Stanujkic et al., 2017), CoCoSo-G (Yazdani et al., 2019a) and COPRAS-G (Zavadskas et al., 2008). In addition to these early studies, these methods have been applied in many different fields in the literature. For applications of the TOPSIS-G method, the studies by Oztaysi (2014), Nyaoga et al. (2016), Zolfani et al. (2012), and Wang et al. (2022) can be considered. Applications of the ARAS-G method can be found in Heidary Dahooie et al. (2018), Chalekaee et al. (2019), and Badi and Elghoul (2023). Studies using the WASPAS-G method include Bakhat and Rajaa (2019) and Halil (2023). For the EDAS-G method, examples are provided by Supçiller and Bayramoğlu (2020), Adali et al. (2022), and Radovanović et al. (2023). In addition, applications of the CoCoSo-G and COPRAS-G methods can be found in Chakraborty et al. (2025) and Yıldırım and Timor (2019), respectively.

Overall, the existing Grey MCDM literature reveals systematic differences in how grey numbers are handled across methods. There are several fundamental differences in how these methods are applied. These differences mainly arise from (i) the stage at which whitening is applied, (ii) the normalization procedures adopted, (iii) the choice of comparison and ranking operators, and (iv) whether structural modifications to the original classical algorithms are required. Although these aspects are discussed in method-specific studies, they are often embedded within algorithmic formulations, which makes cross-method comparison difficult. Most methods developed to apply classical MCDM techniques to grey numbers involve a whitening process at a certain stage. In the classical TOPSIS method (Hwang and Yoon, 1981; Yoon and Hwang, 1995), the Euclidean distance used as a separation measure is calculated based on the weighted decision matrix, whereas in the TOPSIS-G method (Lin et al., 2008; Zavadskas and Turskis, 2010), distances are calculated directly based on the normalized decision matrix using a Minkowski distance-based approach that also incorporates weights. The subsequent steps of TOPSIS-G continue based on these distances obtained as white numbers. Similarly, in ratio-based methods such as ARAS-G and WASPAS-G, whitening is performed just before the comparison stage after the optimality functions are obtained. EDAS-G likewise introduces an appraisal step based on the whitening of grey numbers prior to comparison. Similar examples can be found in other classical MCDM methods adapted to grey numbers, such as CoCoSo-G (Yazdani et al., 2019a) and COPRAS-G (Zavadskas et al., 2008). Another distinction arises among different versions of the same

MCDM method developed for grey numbers. In the case of TOPSIS-G, studies differ in terms of normalization procedures and the calculation of scores used for alternative comparison. Differences in normalization can be clearly observed in the approaches proposed by Lin et al. (2008) and Zavadskas and Turskis (2010). In addition, unlike Lin et al. (2008), Li et al. (2007) and Wang (2009) used the degree of grey possibility and Grey Relational Grade, respectively, to rank alternatives in their grey extensions of TOPSIS.

In summary, the main differences in implementations are mainly related to the normalization procedures and the selection of comparison operators. It can be observed that classical MCDM methods have been adapted to grey numbers through different methodological frameworks, in which arithmetic and comparison operations are often redefined or transformed at certain stages of the algorithms to handle grey information. In contrast to the Grey MCDM variants discussed above, the present study adopts an operator-level perspective rather than modifying the procedural steps of classical MCDM methods. By enabling arithmetic and comparison operators to directly operate on grey numbers, classical MCDM algorithms can be applied without altering their original structure, while subjective decisions related to comparison and ranking are deferred to the final stage of the analysis.

Preliminaries

A grey number is a number whose exact value is unknown, but for which a range within which the value lies is known. A grey number is usually represented by the symbol {⊗}. There are different types and classifications of grey numbers, including grey numbers with only a lower bound, grey numbers with only an upper bound, interval grey numbers, as well as continuous and discrete grey numbers, etc. In applications, a grey number is generally expressed as an interval or a general set of numbers (Liu and Lin, 2006).

In the following, the definitions, rules, and notations of interval grey numbers are presented based on (Liu and Lin, 2006). This study focuses on interval grey numbers, which are defined as follows:

Definition 1: Interval grey numbers ⊗ have both a lower bound a_L and an upper bound a_U, written $\otimes \in [a_L, a_U]$.

Remark: We denote the set of all grey numbers by $R(\otimes)$. We also represent an element of $R(\otimes)$, that is, $\otimes x \in [a, b]$.

Definition 2: When $\otimes \in (-\infty, \infty)$ or $\otimes \in (\otimes_1, \otimes_2)$, that is, when ⊗ has neither a lower bound nor an upper bound, or when both the lower and upper bounds are grey numbers, ⊗ is called a black number.

Definition 3: When $\otimes \in [a, a]$, that is, when the lower and upper bounds are equal, ⊗ is called a white number.

Another key concept associated with grey numbers is the whitenization operation. In cases where the distribution information of an interval grey number is hardly known, the equal weight mean whitenization is typically applied. The formal definitions of these concepts are provided in Definition 4 and Definition 5.

Definition 4: For a general interval grey number $\otimes \in [a, b]$, the corresponding whitened value $\tilde{\otimes}$ is defined as:

$$\tilde{\otimes} = \alpha a + (1 - \alpha)b, \quad \alpha \in [0, 1].$$

Definition 5: In the case of equal weight whitenization, the whitenization value obtained by setting α = 1/2 , is referred to as the equal weight mean whitenization. The basic operations of interval grey numbers have been defined based on Deng (1985) and are widely used in the grey systems literature; the presentation given below follows Liu (2025).

Rule 1. Let $\otimes_1 \in [a, b]$, $a < b$; $\otimes_2 \in [c, d]$, $c < d$. The addition of $\otimes_1$ and $\otimes_2$ is defined as:

$$\otimes_1 + \otimes_2 \in [a + c, b + d]$$

Rule 2. Let $\otimes \in [a, b]$, $a < b$, and k a positive real number. The scalar multiplication k with $\otimes$ is defined as:

$$k \cdot \otimes \in [ka,\ kb]$$

Rule 3. Let $\otimes \in [a, b]$ with $a < b$. The negative inverse of $\otimes$, denoted by $-\otimes$, is defined as follows:

$$- \otimes = [-b, -a].$$

Rule 4. Let $\otimes_1 \in [a, b]$, $a < b$; $\otimes_2 \in [c, d]$, $c < d$. The difference between $\otimes_1$ and $\otimes_2$ is defined as:

$$\otimes_1 - \otimes_2 = \otimes_1 + (- \otimes_2) \in [a - d, b - c]$$

Rule 5. Let $\otimes_1 \in [a, b]$, $a < b$; $\otimes_2 \in [c, d]$, $c < d$.

The multiplication of two interval grey numbers $\otimes_1$ and $\otimes_2$ is defined as:

$$\otimes_1 \cdot \otimes_2 \in [\min\{ac, ad, bc, bd\}, \max\{ac, ad, bc, bd\}]$$

Rule 6. Let $\otimes \in [a, b]$, $a < b$, and k a positive real number, the kth power of $\otimes x = [a, b]$ is defined as:

$$\otimes k \in [ak, bk]$$

Rule 7. Let $\otimes \in [a, b]$, $a < b$, $a \neq 0$, $b \neq 0$, and $ab > 0$, the reciprocal of $\otimes x = [a, b]$ is defined as :

$$(\otimes x)^{-1} \in \left[\frac{1}{b}, \frac{1}{a}\right] \tag{1}$$

Rule 8. Let $\otimes_1 \in [a, b]$, $a < b$; $\otimes_2 \in [c, d]$, $c < d$, $c \neq 0$, $d \neq 0$, and $cd > 0$. The division of $\otimes_1$ by $\otimes_2$ is defined as:

$$\frac{\otimes_1}{\otimes_2} = \otimes_1 \times \otimes_2^{-1} \in \left[\min\left(\frac{a}{c}, \frac{a}{d}, \frac{b}{c}, \frac{b}{d}\right), \max\left(\frac{a}{c}, \frac{a}{d}, \frac{b}{c}, \frac{b}{d}\right)\right] \tag{2}$$

In addition to the operations of addition, subtraction, multiplication, and division for interval grey numbers, there are a number of useful real-valued functions of intervals. Definitions for the center, width, and radius functions are provided below. Several other real-valued functions of intervals have been established and applied in the literature. For a detailed discussion on interval numbers and arithmetic, the works of Moore (1966); Hansen and Walster (2004); Moore et al. (2009) can be referred to.

Definition 6: The length of grey number $\otimes x \in [a, b]$ is defined as:

$$L(\otimes x) = |b - a|$$

It is clear that $L(\otimes x) : R(\otimes) \rightarrow R+$ is a function, as given in (Nasseri et al., 2016).

Definition 7: The whitened midvalue of a grey number, $\otimes x$, is defined as the midpoint value between the lower and upper bounds, and the width of $\otimes x$ is defined as the difference between the upper and lower bounds. Thus, its whitened midpoint (center) xc and width xW are defined as follows, as given in (Huang, 1994):

$$x_c = \frac{1}{2}(b + a), x_W = (b - a)$$

Definition 8: The radius of the interval grey number $\otimes x$, is defined as x_R =$(b-a)/2$, following (Darvishi et al., 2019).

Definition 9: An interval number A is a closed and connected subset of R, denoted by $A \in [a_1, a_2]$. This interval is defined as:

$$A = \{p \in R : a_1 \leq p \leq a_2\}$$

where a_1 and a_2 are the lower and upper boundary values of the interval, respectively. An interval number can alternatively be represented using its center and radius, written as $A = \langle x_c, x_R \rangle$. In fact, any real number $p \in$ R can be considered an interval number $[p, p]$, with center p and radius equal to zero (Bhunia and Samanta, 2014). In the literature, the rules valid for interval numbers are also adopted when examining order relations for interval grey numbers. Several studies, such as Darvishi et al. (2019); Razavi Hajiagha S et al. (2013); Nasseri et al. (2016) etc., can be examined“. Therefore, we have also evaluated these relations for interval grey numbers in the context of Definition 9.

The definitions of interval order relations for comparing intervals in the context of decision-making problems are not unique. There are several different approaches, which will be discussed in detail in Section 4.1.

Remark: In the remainder of the paper, the expression “grey number” will be used to refer to “interval grey number”.

Methodology

An MCDM tool, in the mathematical sense, is a chain of operations which are defined for the sets R → R (e.g. plus, subtraction, multiplication, division, etc.), Rn → Rn (operations on vectors), and Rn → R (summary of vectors, e.g. average, median, standard deviation, norm, etc.), R$n \times m$ → R$j \times k$ (matrix operations, e.g. transpose, inversions, summation, and product of matrices, etc.), R$n \times n$ → R (e.g. determinant, rank, etc.). In that point of view, a decision matrix, a vector of weights, and a list of directions are fed into the TOPSIS method and the operations of standardization, weighting of decision matrix, obtaining the desired and undesired vectors, and finally calculating the scores are applied sequentially by iterations.

Table 1

Operations required to calculate TOPSIS steps

Steps of the Method	Required Operation	Required Operators
Normalization	Vector norm	+, ^, sqrt
Weighting	Vector cross product	×
Desired and undesired vectors	Comparing values	<, ≤, >, ≥
Distance to desired and undesired vectors	Euclidean distance	-, +, ^, sqrt
Obtaining Scores	$\frac{d_i^-}{d_i^- + d_i^+}$	+, /
Obtain the best alternative	Comparing values	<, ≤, >, ≥

Table 1 summarizes the operators required by TOPSIS method. The set of arithmetic operators +, −, ×, /, ∧, and √ are well-defined for the real set R. The comparison operators <, ≤, >, and ≥ are also defined for the real numbers. In general, TOPSIS is applicable for any set of numbers if the required arithmetic and comparison operators are defined.

Note that the arithmetic operations required by the TOPSIS method is defined in Section 3, as a result, any vector and matrices of Grey numbers are operable using this operations, however, some of the comparison operators should be defined in order to perform the other steps including obtaining the desired/undesired vectors and sorting the scores. Since it is beyond the interval arithmetic, a more subjective decision making progress is required to define comparison operators in higher dimensional spaces such as the one that Grey numbers are defined on.

Several definitions of comparison operators in the literature will be summarized in Section 4.1, and it was also emphasized that there is potential for the invention of many new meaningful comparison operators in multi-dimensional spaces.

Definition of comparison operators on Grey numbers

The comparison relations between interval grey numbers are of fundamental importance for solving decision problems. Ironically and recursively, definition of < operator is an other MCDM problem, that is, supposing $\otimes x \in [a_1, b_1]$ and $\otimes y \in [a_2, b_2]$ and testing whether $\otimes x < \otimes y$ can be represented as a decision table as show in Table 2.

Table 2

Comparison of grey numbers

	Lower bound	Upper bound
Weights	w_1	w_2
Directions	min	min
$\otimes x$	a_1	b_1
$\otimes y$	a_2	b_2

It is shown in Table 2 that the problem is an MCDM problem and the alternative with the higher score is less than the other. It is expected that the dominant alternative will always gain the highest score, whereas, if there is a conflict between the components, the score will change depending on the algorithm or method selected. Let's consider $\otimes x$ and $\otimes y$ forma pair of arbitrary grey numbers represented by intervals: $\otimes x \in [a1, b1]$, $\otimes y \in [a2, b2]$. When it comes to equality, it can be easly said that the Grey numbers $\otimes x$ and $\otimes y$ equal to each other if $a1 = a2$ and $b1 = b2$, however, it is not that easy to judge on whether a Grey number is less than the other or not. Defining a total order relation for interval grey numbers is not as simple as it is for real numbers. Therefore, many researchers have developed different comparison metrics based on various mathematical foundations. In the existing literature, the definitions of ordering relations are developed based on set properties, fuzzy applications, probabilistic approaches, value-based approaches, or specific indices or functions. In most studies, there is a fundamental classification used for interval numbers. There are three different types of intervals: Non-overlapping intervals (Type I), Partially overlapping intervals (Type II), and Completely overlapping intervals (Type III) (Karmakar and Bhunia, 2012). Figures 1 - 3 represent Type-I, Type-II, and Type-III intervals, respectively[1].

[1]Adapted from (Karmakar and Bhunia, 2012)

Figure 1
Type - I Intervals

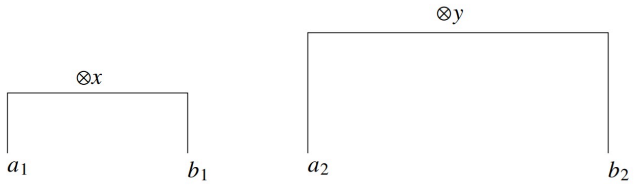


Figure 2
Type - II Intervals

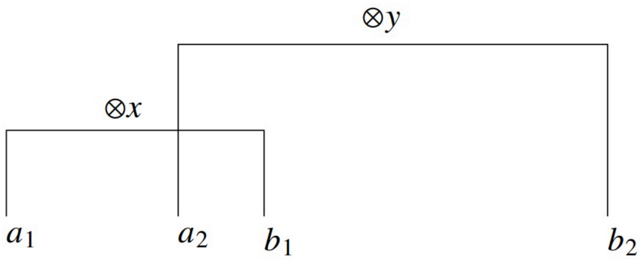
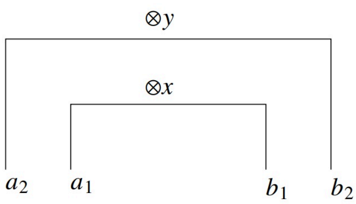


Figure 3
Type - III Intervals



Most of the definitions of order relations differ according to the types of intervals given. The choice of the ordering relation to be used can also vary depending on whether the decision problem is focused on maximization or minimization, as well as the subjective characteristics of the decision maker. Some approaches to order relations are introduced below.

For readability, all definitions are presented in a unified notation.

Moore's approach

The foundational work in this regard, Moore (1979) defines the order relations between two grey interval numbers $\otimes x \in [a_1, b_1]$ and $\otimes y \in [a_2, b_2]$ as follows:

$$\otimes x < \otimes y \iff b_1 < a_2$$

$$\otimes x \subseteq \otimes y \iff a_1 \leq a_2 \text{ and } b_1 \leq b_2$$

Moore (1979) also defined the equality of two intervals as follows:

$$\otimes x = \otimes y \iff a_1 = a_2 \text{ and } b_1 = b_2$$

This definition indicates that two grey intervals are equal if and only if their corresponding endpoints are equal. The first order relation given by Moore (1979) is applicable for non-overlapping intervals. The second relation is an extension of the subset definition for intervals.

Hu and Wang's approach

Hu and Wang (2006) proposed a novel ordering relation for interval numbers and showed that it satisfies reflexivity, anti-symmetry, transitivity, and comparability. Their interval ordering relation, which is based on the center and radius of interval grey numbers, is presented below, following Bhunia and Samanta (2014): For any two interval grey numbers $\otimes x \in [a_1, b_1]$ and $\otimes y \in [a_2, b_2]$, let x_c and y_c be the centers, let x_R and y_R be the radius of $\otimes x$ and $\otimes y$, respectively. Thus,

$$\otimes x \preceq \otimes y \Leftrightarrow \begin{cases} x_c < y_c \text{ if } x_c \neq y_c \\ x_R \geq y_R \text{ if } x_c = y_c \end{cases}$$

$$\otimes x \prec \otimes y \Leftrightarrow \otimes x \preceq \otimes y \text{ and } \otimes x \neq \otimes y$$

There are order relation approaches in the literature proposed for different types of intervals and different decision maker characteristics.. For these approaches, the studies of Mahato and Bhunia (2006) and Sahoo et al. (2012) can be examined. Bhunia and Samanta (2014) proposed a simplifying order interval that generalizes the definitions of Sahoo et al. (2012) for all types of intervals.

Bhunia and Samanta's approach

Bhunia and Samanta (2014) defines order relations with the concepts of center and radius as defined as follows. The order relation ≥max between two interval grey numbers $\otimes x \in [a_1, b_1]$ and $\otimes y \in [a_2, b_2]$, let x_c and y_c are centers, let x_R and y_R are radius of $\otimes x$ and $\otimes y$, respectively. Thus,

$$\otimes x \underset{\max}{\geq} \otimes y \Leftrightarrow \begin{cases} x_c > y_c \text{ if } x_c \neq y_c \\ x_R \leq y_R \text{ if } x_c = y_c \end{cases}$$

$$\otimes x \underset{\max}{>} \otimes y \Leftrightarrow \otimes x \underset{\max}{\geq} \otimes y \text{ and } \otimes x \neq \otimes y$$

Similarly, the less than counterpart can be defined as follows:

$$\otimes x \underset{\min}{\leq} \otimes y \Leftrightarrow \begin{cases} x_c < y_c \text{ if } x_c \neq y_c \\ x_R \leq y_R \text{ if } x_c = y_c \end{cases}$$

$$\otimes x \underset{\min}{<} \otimes y \Leftrightarrow \otimes x \underset{\min}{\leq} \otimes y \text{ and } \otimes x \neq \otimes y$$

More on comparisons in the multi-dimensional space

It is easy to catch that more operators can be defined in the same space, and it is clear that these new operators will be at least as meaningful as the others. In a multi-objective maximization problem, supposing the objective functions are $f_1, f_2 \ldots, f_n$, and x_1, x_2 are two candidate solutions, x_1 is better than x_2, e.g. $x_1 > x_2$, if and only if there is at least a single i that satisfies $f_i(x_1) > f_i(x_2)$ and $f_j(x_1) \geq f_j(x_2)$ for $j \in 1 \ldots n$ /i. If the conditions are held, then x_2 is dominated by x_1 and this dominance measure is widely used in a sorting algorithm called non-dominated sorting algorithm, which as a part of the multi-objective optimization (Deb, 2011). Using the dominance rule given above, the comparison operator < can be defined to compare two Grey numbers. Supposing $\otimes x \in [a_1, b_1]$ and $\otimes y \in [a_2, b_2]$, it can be written that

$$\otimes x < \otimes y = \begin{cases} \text{true if } a_1 < a_2 \\ \text{true if } a_1 = a_2 \wedge b_1 < b_2 \\ \text{false otherwise} \end{cases}$$

Note that the > and ≥ operators can be easily defined using a negation, as they are the complements of the ≤ and < operators, respectively.

Operator overloading, multiple dispatch, and implementations

Many programming languages allow users to define multiple versions of methods and functions with different types of arguments. This concept is generally referred to as operator overloading and method overloading. Operators primitively work on built-in data types such as integers and floating-point numbers can be redefined to work on user-defined data types such as complex numbers, fractions, vectors, and matrices (Yang, 2001; Stroustrup, 1984). Compiled languages such as C++ handle the implementation of this concept at compile time. However, Julia introduces a new concept called multiple dispatch. Similarly, the Julia compiler decides at runtime which version of a function to compile and use, and the compiled instances are called methods of the function (Bezanson et al., 2018; Gowda et al., 2022).

For instance, when 4 + 3 is called in Julia, the result is of type Int64, whereas the result of 4.0 + 3.0 is of type Float64. This means that the + operator behaves differently depending on the types of its arguments. The same + operator is also defined for other integral numeric types, as well as for number systems with multiple components, such as Rational and Complex numbers.

Since vectors and matrices are first-class citizens in Julia, if a set of operators is defined for a specific numeric type, then vectors and matrices of that type are also operable. This feature offers infinite possibilities, including allowing the use of predefined linear algebra tools and building a complete numerical mathematics framework on user-defined numbers, such as Grey numbers. Suppose that the GreyNumber is a Julia struct with two fields and is defined as follows:

```
struct GreyNumber
    a::Float64,
    b::Float64,
end
```

The plus operator can be overridden to add two Grey numbers as defined below:

```
function Base.:+(g1::GreyNumber, g2::GreyNumber)::GreyNumber
GreyNumber(g1.a + g2.a, g1.b + g2.b)
end
```

Similarly, the product of two Grey numbers can be defined as follows:

```
function Base.:*(g1::GreyNumber, g2::GreyNumber)::GreyNumber
   v = [g1.a * g2.a, g1.a * g2.b, g1.b * g2.a, g1.b * g2.b]
   return GreyNumber(minimum(v), maximum(v))
end
```

Once the plus and product operators are defined, those operations for vectors containing Grey numbers are also automatically defined. Recall that the Table 1 summarizes the operators defined for grey numbers which are required by TOPSIS method. If the Julia counterparts of the required functions are defined once, the classical TOPSIS implementation should just work as it works with real numbers thanks to the multiple dispatch feature of Julia. Similarly, a ≤ redefinition for grey numbers can be defined as follows:

```
function Base.:<=(g1::GreyNumber, g2::GreyNumber)::Bool
ac, aw = center(g1), radius(g1)
bc, bw = center(g2), radius(g2)
if ac != bc
return ac < bc
else
return aw <= bw
end
end
```

This function compares two grey numbers as Julia does for real numbers. Supposing x and y are two distinct grey numbers, the following code will work as expected:

```
julia> x = GreyNumber(4, 5)
julia> y = GreyNumber(2, 3)
julia> x <= y
false
```

Note that the comparator operator is the Julia implementation of the ≤ operator suggested by Bhunia and Samanta (2014) and the other operators can be defined in a similar way. All of the examples given in this paper use the same implementation of the given definitions. 4.3. Whitenization versus Overloading Operators Grey numbers are used for modeling uncertainties. In statistics, if two independent random variables X and Y follow a Gaussian distribution with mean μ and standard deviation σ then the new random variable $W = X - Y$ also follows the same distribution rather than zero. Similarly, supposing a Grey number is $\otimes x \in [1, 4]$, the subtraction $\otimes x - \otimes x$ is $\otimes x + (-1 \times \otimes x)$, that is

$$\otimes (1,4) - \otimes (1,4) = \otimes (1,4) + \otimes (-4,-1) = \otimes (-3,3)$$

Which is apparently non-zero. This also means that the Euclidean distance between a grey vector and itself is not zero.

Figure 4

4 Grey Numbers in 2D space

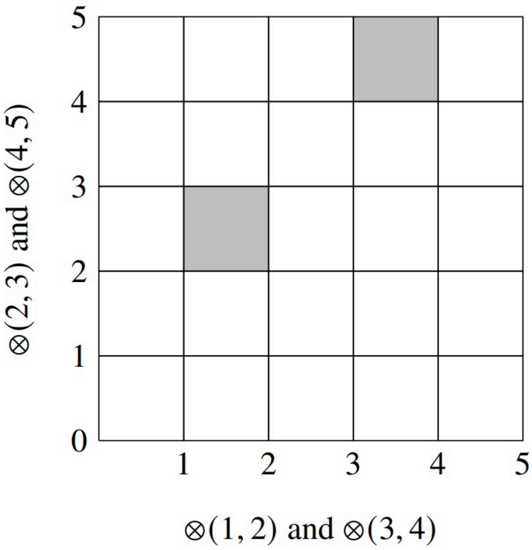


Figure 4 represents 4 Grey numbers in the 2-dimensional Euclidean space. The grey numbers ⊗(1, 2), ⊗(3, 4) are represented on the horizontal axis, whereas ⊗(2, 3) and ⊗(4, 5) are located on the vertical axis. Since Grey numbers span line segments, their intersections are represented as rectangles. It is expected that the distance between two points is a real number, however, distance between two rectangles is undefined and it depends on the selected corner. When a corner is not selected, the distance between two rectangles is uncertain which is modeled by a Grey number. Supposing the grey numbers in the horizontal axis form a vector, namely v, and grey numbers in the vertical axis form an other vector, namely u then the squared Euclidean distance between u and v is

$$D(u,v) = (\otimes\,(1,2) - \otimes\,(2,3))^2 + (\otimes\,(3,4) - \otimes\,(4,5))^2$$

Which is ⊗(0.0, 7.997), a distance measure in a Grey space again, modeling the uncertain distance between two geometric objects. If we whiten the result for α = 0.5, then we obtain

$$\mathrm{White}(\otimes\,(0.0, 7.997)) = 3.998$$

is exactly equals to the Euclidean distance between whitenized u and v.

Things change when the sample data is generated without using a pattern, that is, suppose we have two grey vectors u and v and they are defined as

$$u = \begin{bmatrix} \otimes\,(0.718, 0.818) \\ \otimes\,(0.267, 0.786) \end{bmatrix}$$

$$\text{and}$$

$$v = \begin{bmatrix} \otimes\,(0.425, 0.660) \\ \otimes\,(0.656, 0.877) \end{bmatrix}$$

and the squared Euclidean distance between u and v is ⊗(−0.075, 0.527), whereas, the squared Euclidean distance between whitened u and v is 0.108 for whitening parameter of α = 0.5. However, the α value needed to be chosen as 0.225 for the distance obtained from Grey numbers to be equal to the distance obtained from whitened numbers. Therefore, the choice of α in the whitening operator also affects the results in MCDM methods if the decision matrix is formed by Grey numbers.

Experimental results

In this section, we will demonstrate the applicability of the TOPSIS method with pseudo-grey numbers using multiple dispatch. Note that the results are obtained using a single implementation of the method but the operators are dispatched for the current data type.

Illustrative Examples

TOPSIS with Pseudo-Grey Numbers Using Operator Overloading

Table 3 represents a decision problem with 3 alternatives and 4 criteria. The weights of the criteria are 1 4 for each criterion and the direction of the criteria is max for all criteria.

Table 3

A sample decision matrix

	w_1	w_2	w_3	w_4
dir	max	max	max	max
A_1	0.538	0.070	0.005	0.497
A_2	0.997	0.115	0.369	0.319
A_3	0.0275	0.905	0.627	0.559

The classical TOPSIS method produces the scores for the alternatives as follows:

$$\text{Scores} = \begin{bmatrix} 0.275 \\ 0.504 \\ 0.600 \end{bmatrix} \tag{3}$$

Now we transform the original problem with using pseudo-grey numbers and the new decision matrix is shown in Table 4. Note that the transformed problem has grey numbers with a same lower bound and upper bound which are inherently taken from the original problem. The same TOPSIS method is applied without changing the implementation. Most of the work is done by the operator overloading, a.k.a, multiple dispatch in Julia.

Table 4

A sample decision matrix with pseudo-grey numbers

	w_1	w_2	w_3	w_4
dir	max	max	max	max
A_1	$\otimes(0.538, 0.538)$	$\otimes(0.070, 0.070)$	$\otimes(0.005, 0.005)$	$\otimes(0.497, 0.497)$
A_2	$\otimes(0.997, 0.997)$	$\otimes(0.115, 0.115)$	$\otimes(0.369, 0.368)$	$\otimes(0.319, 0.319)$
A_3	$\otimes(0.0275, 0.275)$	$\otimes(0.905, 0.905)$	$\otimes(0.627, 0.627)$	$\otimes(0.559, 0.559)$

The classical TOPSIS method that operates on grey numbers produces the scores for the alternatives as follows:

$$\text{Scores} = \begin{bmatrix} \otimes(0.275, 0.275) \\ \otimes(0.504, 0.504) \\ \otimes(0.600, 0.600) \end{bmatrix}$$

Which are the same as the scores obtained from the original problem as the grey numbers are transformed with the same lower bound and upper bound. This is a proof of concept that the TOPSIS method can be applied to grey numbers without changing the implementation of the steps of the method.

TOPSIS with Real Grey Numbers Using Operator Overloading

Table 5 represents a decision problem with 6 alternatives and 5 criteria. The weights of the criteria are 1 / 5 for each criterion and the direction of the criteria is max for all criteria. Differently from Table 4, the grey numbers have different lower bounds and upper bounds, so we call them real-grey numbers. The classical TOPSIS method that operates on real-grey numbers produces the scores for the alternatives as follows:

$$\text{Scores} = \begin{bmatrix} \otimes(0.211, 0.857) \\ \otimes(0.118, 1.202) \\ \otimes(0.228, 0.981) \\ \otimes(0.187, 0.910) \\ \otimes(0.201, 1.071) \\ \otimes(0.160, 1.101) \end{bmatrix}$$

Table 5

A sample decision matrix with real-grey numbers

	w_1	w_2	w_3	w_4	w_5
	1/5	1/5	1/5	1/5	1/5
dir	max	max	max	max	max
A_1	$\otimes(0.538, 0.997)$	$\otimes(0.028, 0.070)$	$\otimes(0.115, 0.906)$	$\otimes(0.005, 0.369)$	$\otimes(0.123, 0.531)$
A_2	$\otimes(0.497, 0.627)$	$\otimes(0.320, 0.560)$	$\otimes(0.641, 0.927)$	$\otimes(0.379, 0.427)$	$\otimes(0.276, 0.310)$
A_3	$\otimes(0.356, 0.377)$	$\otimes(0.606, 0.839)$	$\otimes(0.075, 0.716)$	$\otimes(0.062, 0.577)$	$\otimes(0.728, 0.812)$
A_4	$\otimes(0.215, 0.812)$	$\otimes(0.115, 0.225)$	$\otimes(0.369, 0.997)$	$\otimes(0.0275, 0.275)$	$\otimes(0.198, 0.456)$
A_5	$\otimes(0.127, 0.546)$	$\otimes(0.267, 0.786)$	$\otimes(0.005, 0.497)$	$\otimes(0.538, 0.538)$	$\otimes(0.345, 0.678)$
A_6	$\otimes(0.871, 0.997)$	$\otimes(0.115, 0.309)$	$\otimes(0.369, 0.874)$	$\otimes(0.319, 0.595)$	$\otimes(0.123, 0.544)$

The generated scores are obtained from the classical TOPSIS method without any modification on the implementation of the method. Since the whitenization operator is not used, the scores are grey numbers. We previously mentioned that subjective intervention is needed at some point in the ranking of alternatives. Thanks to the direct application of the classical method to grey numbers, this intervention has been deferred to the final stage. At this point, the researcher may use any subjective ranking algorithm or apply a whitening operator for a specific alpha value. However, the key point emphasized here is that the classical method has become directly applicable to a newly defined data type.

Depending on the selected α value in the whitening process the effect of the lower and upper bounds of grey scores can be controlled. Table 6 represents the results of a sensitivity analysis conducted by varying the α value from 0.0 to 1.0 in increments of 0.1.

Table 6

Rankings of alternatives for different α values

α	A_1	A_2	A_3	A_4	A_5	A_6
0.0	6	1	4	5	3	2
0.1	6	1	4	5	3	2
0.2	6	1	4	5	3	2
0.3	6	1	4	5	3	2
0.4	6	1	4	5	3	2
0.5	6	1	4	5	2	3
0.6	6	1	4	5	2	3
0.7	5	3	2	6	1	4
0.8	4	5	1	6	2	3
0.9	3	6	1	4	2	5
1.0	2	6	1	4	3	5

Table 6 shows the rankings of the alternatives for different α values used in the whitening process. It is observed that several α values up to 0.7 yield the same rankings. However, as α increases, the rankings change significantly. This observation highlights the importance of selecting an appropriate α value when applying the whitening in the context of grey numbers. The choice of α can have a substantial impact on the final rankings of alternatives, emphasizing the subjective nature of this decision-making process.

Classical ARAS, EDAS, and WASPAS methods can also be directly applied to grey numbers with the same methodology. For the demonstration purpose, we applied these methods to the same decision matrix given in Table 5. The rankings of the alternatives obtained from different MCDM methods are summarized in Table 7.

Table 7

Rankings of alternatives obtained from different MCDM methods

Alternative	TOPSIS	ARAS	EDAS	WASPAS
A_1	6	5	6	6
A_2	1	4	4	2
A_3	4	1	1	1
A_4	5	6	5	5
A_5	2	3	2	4
A_6	3	2	3	3

Note that the comparison of the methods in Table 7 is not intended to evaluate the performance of the methods but to demonstrate that classical MCDM methods can be directly applied to grey numbers using operator overloading. The rankings of the alternatives differ across the methods, which is expected due to the different underlying principles and calculations of each MCDM method. This further emphasizes the flexibility and adaptability of the operator overloading approach in handling grey numbers within various decision-making frameworks.

Comparison with other Grey TOPSIS methods

In this section we compare the results obtained from some previous Grey TOPSIS methods with the results obtained from the classical TOPSIS. Zavadskas et al. (2010) provides a case study of contractor selection with five alternatives and six criteria. It is reported that the ranking of the alternatives is

$$[A1 \succ A2 \succ A3 \succ A5 \succ A4]$$

When the Grey TOPSIS method proposed by Zavadskas et al. (2010) is applied to the problem.We applied the classical TOPSIS method on the same problem using grey numbers and obtained the following grey scores for the alternatives:

```
julia> sort(result.scores)
5-element Vector{Any}:
GreyNumber(0.26078084132844775, 0.6334013060970956)
GreyNumber(0.23527302017921461, 0.8134740993812873)
GreyNumber(0.19531593446952664, 0.8730782901987493)
GreyNumber(0.19801742033591807, 0.966628808962525)
GreyNumber(0.1971487406801051, 0.9792626439442739)
```

Since the operators ≥ and ≤ are defined for grey numbers, any operations related to sorting can be directly applied. The ranking of the alternatives is obtained as follows:

```
julia> invperm(sortperm(result.scores, rev = true))
5-element Vector{Int64}:
1
2
3
5
4
```

Which is exactly the same as the ranking reported by Zavadskas et al. (2010). This comparison demonstrates that the classical TOPSIS method, when applied to grey numbers using operator overloading, can yield results consistent with those obtained from specialized Grey TOPSIS methods. Lin et al. (2008) applied a Grey TOPSIS method on a subcontractor selection example of an engineering corporation to demonstrate the feasibility and practicability of the their proposed method. The problem involves three periods, four alternatives, and four criteria. In the problem, there are three decision matrices corresponding to three periods. The scores obtained for each single period are summed to obtain the final scores for the alternatives. They weighted the first period scores, second period scores, and third period scores by 0.5, 0.3, and 0.2, respectively. They reported that the ranking of the alternatives is

$$[A3 \succ A2 \succ A4 \succ A1]$$

When their Grey TOPSIS method is applied to the problem. For the comparison purpose, we applied the classical TOPSIS method on the same problem using grey numbers and obtained the following weighted grey scores for the alternatives:

```
4-element Vector{GreyNumber}:
GreyNumber(0.2589124235097974, 0.8095660204119177)
GreyNumber(0.2223210899691938, 0.9974567307244757)
GreyNumber(0.18213176316314034, 1.0461250681989414)
GreyNumber(0.24093988887916834, 0.9325028158805556)
```

The ranking of the alternatives is obtained as follows:

```
julia> invperm(sortperm(totscores, rev = true))
4-element Vector{Int64}:
4
2
1
3
```

Which is totally different from the ranking reported by Lin et al. (2008). When we whiten the scores with different α values, we obtain different rankings. The results of this sensitivity analysis are shown in Table 8.

Table 8

Rankings of alternatives for different α values

α	A1	A2	A3	A4
0.0	4	2	1	3
0.1	4	2	1	3
0.2	4	2	1	3
0.3	4	2	1	3
0.4	4	2	1	3
0.5	4	2	1	3
0.6	4	1	2	3
0.7	4	1	3	2
0.8	3	2	4	1
0.9	1	3	4	2
1.0	1	3	4	2

In Table 8, it is observed that the rankings of the alternatives change significantly as α changes. For α = 0.8, the ranking obtained from the classical TOPSIS method matches the ranking reported by Lin et al. (2008). This observation highlights the sensitivity of the rankings to the choice of α in the whitening process. The proposed approach leaves the selection of α to the decision maker at the final stage, if needed. Direct use of grey scores without whitening is also an option, as demonstrated in the previous example.

This comparison illustrates a key advantage of the proposed approach. When the classical TOPSIS method is applied to grey numbers without any modifications to the implementation, the analysis produces grey scores that retain the uncertainty information. They are comparable in their grey form, whereas, the user can prefer to whiten the scores with a chosen level of α to obtain crisp scores for ranking. It is also possible to select many α values and observe the changes in rankings to understand the sensitivity of the decision-making process to the whitening parameter. When there is an exact dominance among grey scores, changes in α will not affect the rankings. However, when alternatives are close to each other, different α values may lead to different rankings. In short, the proposed approach provides more flexibility and transparency in handling grey numbers in MCDM methods compared to existing Grey MCDM methods.

Conclusion and Discussion

The paper presents a systematic approach to applying classical MCDM methods to grey numbers through the concept of operator overloading and multiple dispatch in the Julia programming language. The key contribution of this work lies in demonstrating that classical MCDM methods can be directly applied to grey numbers without requiring any modifications to the core algorithms. This is achieved by defining appropriate arithmetic and comparison operators for grey numbers that seamlessly integrate with the existing MCDM methods. It is shown that the classical TOPSIS, ARAS, WASPAS, and EDAS methods become direclty applicable to grey numbers using this approach without any changes to their implementations thanks to operator overloading, a.k.a, multiple dispatch in Julia.

The study highlights that the choice of comparison operators for grey numbers is inherently subjective, reflecting the very nature of the decision-making problem that MCDM methods aim to solve. By deferring this subjective intervention to the final stage of the analysis, researchers gain more flexibility in choosing the most appropriate ranking method for their specific context. This approach contrasts with existing Grey MCDM methods that often incorporate subjective elements during the calculations, such as normalization or whitening steps.

The experimental results confirm that the classical TOPSIS, ARAS, WASPAS, and EDAS methods can be applied to grey numbers without any modifications to its implementation. This is a significant finding as it suggests that other classical MCDM methods could potentially be adapted to grey numbers using the same approach. In this study, we show that the original TOPSIS yields identical grey scores when the decision matrix is composed of grey numbers with identical lower and upper bounds. This example demonstrates the consistency of the proposed approach. When the decision matrix contains real grey numbers with different lower and upper bounds, the original TOPSIS produces grey scores that retain the uncertainty information.We compare our methodology with previous Grey TOPSIS methods and find that our approach can replicate the results of some existing methods while providing additional flexibility in handling grey scores.

Since the propose approach retains the uncertainty from the very beginning to the end of the analysis, it allows for a more transparent decision-making process. The decision maker is able to see how different choices of α affect the final rankings, thus providing insights into the sensitivity of the decision-making process to the handling of uncertainty.

The paper also addresses the issue of whitening parameter selection, showing that different values of the whitening parameter α can lead to different results. This highlights the importance of carefully choosing the whitening parameter when comparing results between grey and whitened analyses.

The proposed approach offers several advantages over existing Grey MCDM methods. First, it maintains the mathematical rigor of classical MCDM methods while extending their applicability to grey numbers. Second, it provides a clear separation between the objective calculation steps and the subjective ranking steps, making the decision-making process more transparent. Third, it leverages modern programming language features (multiple dispatch) to implement the approach in an elegant and maintainable way.

This study provides important contributions to the Grey MCDM literature from both theoretical and practical perspectives. From a theoretical point of view, it is shown that classical MCDM methods can be applied to grey numbers without making any changes to their core mathematical structures, thus preserving their theoretical consistency. In this way, decision-making processes can be handled in a more transparent and systematic manner. From a practical perspective, the proposed approach allows existing classical MCDM algorithms to be used directly and flexibly with grey data, offering an easy-to-apply solution for practitioners. In addition, it is expected that this approach can provide additional flexibility for researchers when dealing with applied studies that involve uncertain data represented by grey numbers. Moreover, by relying on modern programming language features, the proposed implementation increases the reproducibility of the results and the applicability of the approach to real-world decision-making problems. Overall, this study makes a meaningful contribution to the literature on decision-making problems involving uncertainty and incomplete information.

Despite the contributions of this study, it has some limitations that can be addressed in future research. Future work could explore the extension of this approach to other classical MCDM methods beyond TOPSIS, ARAS, EDAS, and WASPAS. Since we mainly focused on demonstrating the direct applicability of classical MCDM methods to grey numbers, further studies could investigate the performance and robustness of these methods when applied to grey data in various decision-making scenarios. Exploring different comparison operators for grey numbers and their impact on the results of MCDM methods could be another avenue for future research. It is also worth investigating the integration of this approach with other types of uncertain data representations, such as fuzzy numbers, to assess its versatility and adaptability. Additionally, future research could focus on developing guidelines for selecting appropriate whitening parameters based on the specific characteristics of the decision-making problem at hand. Finally, empirical studies involving real-world decision-making problems could be conducted to validate the effectiveness and practicality of the proposed approach in various application domains.

The paper concludes by emphasizing that while the application of classical MCDM methods to grey numbers is now more straightforward, the fundamental challenge of dealing with uncertainty and subjectivity in decision-making remains. The proposed approach provides a more systematic way to address this challenge while maintaining the mathematical integrity of the underlying methods.

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